Journal
of
The Franklin Institute

EDITOR, HENRY BUTLER ALLEN, MET.E., D.Sc
ASSISTANT EDITOR, JOHN FRAZER, PH.D.

Associate Editors:

WILDER D. BANCROFT, PH.D.
C. B. BAZZONI, PH.D.
ARTHUR L. DAY, SC.D.
R. EKSERGIAN, PH.D.
A. S. EVE, F.R.S.
PAUL D. FOOTE, PH.D.
W. J. HUMPHREYS, PH.D.
C. E. K. MEES, D.SC.
WILLIAM B. MELDRUM, PH.D.
HENRY C. SHERMAN, SC.D.
W. F. G. SWANN, D.SC.
HUGH S. TAYLOR, D.SC
A. F. ZAHM, PH.D.
JOHN ZELENY, PH.D.

Committee on Publications:

G. H. CLAMER, CHAIRMAN
RICHARD HOWSON
C. L. JORDAN
LIONEL F. LEVY
MALCOLM LLOYD, JR.
HIRAM S. LUKENS
RICHARD H. OPPERMANN
WINTHROP R. WRIGHT

Vol. 242 **OCTOBER, 1946** **No. 4**

CONTENTS

The Relay Oscillator and Related Devices ..... RONALD L. IVES 243

Application of Circuit Theory to the Design of Servomechanisms .... ALBERT C. HALL 279

The Maximum Inertia Force and Maximum Speed of the Crosshead in Reciprocating Engines ..... W. S. KIMBALL 309

Notes from the National Bureau of Standards 325

The Franklin Institute Notes 333

Notes from the Biochemical Research Foundation 335
A Reprint Collection as an Easily Accessible Source of Reference Material
ANNE HIMMELBERGER LONGENBACH

Book Reviews 337

Publications Received 338

Current Topics 278, 308, 339

Published by
THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA
Prince and Lemon Streets, Lancaster, Penna., and
Benjamin Franklin Parkway at Twentieth St., Philadelphia 3, Penna.

SIX DOLLARS PER YEAR (Foreign Postage Additional) **SINGLE NUMBERS, SIXTY CENTS**
Indexes to the semi-annual volumes of the JOURNAL are published with the June and December numbers

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 242 OCTOBER, 1946 No. 4

THE RELAY OSCILLATOR AND RELATED DEVICES.*

BY

RONALD L. IVES.

ABSTRACT.

Several arrangements of relay, condenser, and battery can be made to oscillate dependably. The electrical properties of these oscillators, as well as of a number of derivative devices, are here outlined.

INTRODUCTION.

About two generations ago, numerous electrical experimenters found that the tone of a buzzer of conventional design could be altered by shunting a condenser across either the coil or the contacts. At about the same time, it was discovered that a telegraph relay could be made to "copy behind" by use of a series resistor and a shunt condenser.

In the early 1930s, radio service men, seeking to eliminate radio interference from buzzers and doorbells, connected large electrolytic condensers across the contacts of these annunciators, and found that the tone was markedly reduced in frequency. These findings were seldom reported beyond the local radio club.

Since large electrolytic condensers became commercially available about fifteen years ago, some applications of the "electric ticker," "tuned buzzer," and "lagged relay" have been made in the communications industry. Most of the information concerning these devices never found its way into professional literature, but was passed about, by word of mouth, among experimenters and engineers.

THEORETICAL OUTLINE.

Salient Properties of a Series Circuit. When a battery, condenser and electromagnet (such as a relay) are connected in series, as in Fig. 1, the current will be a maximum very soon after the circuit is closed, and will slowly decline toward zero, theoretically reaching it in infinite time, actually becoming zero in a matter of minutes at most. When a com-

* Published by permission of the Chief, Chemical Warfare Service, U. S. A.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

mercial electrolytic condenser [1] is used, the current asymptote will be the leakage current. A typical curve is shown in Fig. 2. The rounding of the initial peak, which should be sharp, is in large part due to instrument damping. Theoretical peak is 10.87 milliamperes; recorded peak 9.90 ma.

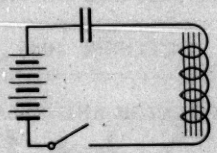


FIG. 1. Simple series circuit.

In a battery-condenser-resistor circuit with "pure" components, the initial current, I_0, is shown by Ohm's Law, a completely discharged condenser acting as a "dead short."

$$I_0 = \frac{E}{R}, \tag{1}$$

where: [2]

I_0 = initial current (amperes),
E = applied voltage (volts),
R = resistance (ohms).

Current in the circuit, at any time after initial closure, assuming pure components, is:

$$I = \frac{E}{R} e^{-T/RC}, \tag{2}$$

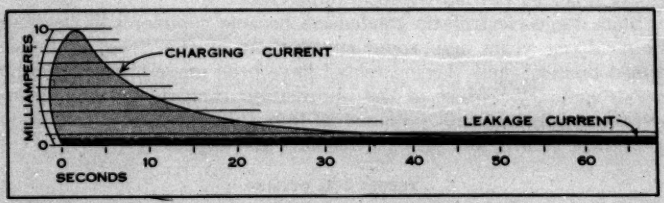


FIG. 2. Current flow in a series circuit (Fig. 1) when electrolytic condenser is used. Were the instrument response instantaneous, and the relay coil a pure resistance, the initial peak would be sharp. Voltage here was 12, capacity 18,000 microfarads (nominal) and circuit resistance 1,104 ohms.

[1] Properties of commercial electrolytic condensers are outlined in Terman, F. E., "Electrical Engineers Handbook," New York, 1943, 126–128.

[2] The same symbols, in all equations used in this paper, refer to the same quantities and units wherever used. Only the new symbols used in each equation will be defined below it.

where:

I = Current at any instant (amperes),
e = Base of Napierian system of logarithms,
C = Capacity of condenser (farads),
T = Time (seconds) after circuit closure.

As this paper is concerned with time functions of relay-condenser devices; and as most engineering computations use common logarithms, the following conversion is made:

$$\text{Log}_e I = \log_e \frac{E}{R} - \frac{T}{RC},$$

$$T = RC \log_e \frac{E}{RI}, \tag{3}$$

$$(\log_e N = 2.303 \log_{10} N),$$

$$T = 2.303RC \log_{10} \frac{E}{RI}.$$

By arranging the contacts of the relay of Fig. 1 so that the condenser is discharged whenever it approaches full charge (i.e., the inflow current falls to the drop-out value of the relay, which must exceed the condenser leakage current), an oscillator operating on the "Sisyphian Principle" is obtained (Fig. 3).

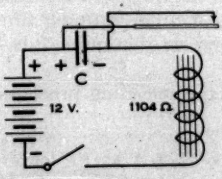


FIG. 3. Simplest form of relay oscillator, consisting of battery, condenser, and relay, in series.

Were all circuit components "pure," and the relay armature travel time zero, the time, T_c, for one cycle would be shown by:

$$T_c = 2.303RC \log_{10} \frac{E}{RI_d}, \tag{4}$$

where: I_d = drop-out current of relay (amperes).

With commercial electrical equipment, no component is "pure." A relay coil, although rated in ohms, has distributed capacity and inductance. The former, measurable in micro-microfarads, is negligible where the controlling capacity is rated in tens, hundreds, or thousands of microfarads.

The relay coil inductance may be appreciable, the impedance of some commercial relay coils at 60 cycles being from two to five times their resistance. In this "*LCR*" series circuit, three conditions—aperiodic current, critical damping, and damped oscillations—are possible. A relay oscillator will function dependably under the first two conditions, but will develop "a will of its own" under the third.

Rough computation shows that the inductance (in Henries) of a commercial relay coil seldom exceeds 1/60th of its resistance (in Ohms). In a series "*LCR*" circuit, oscillatory current will not be present if:

$$\left[\frac{R^2}{4L^2} - \frac{1}{LC}\right] \quad \text{is zero or positive.}^3$$

Substituting for L, the inductance in Henries, its maximum probable equivalent, $R/60$, this discriminant becomes:

$$15 - \frac{1}{RC}. \tag{5}$$

Hence, damped oscillations will not be present, in most instances where commercial equipment is used, in a relay oscillator where 15 exceeds $1/RC$. If $1/RC$ exceeds 15, the oscillator may have two frequencies, not necessarily related: one (unwanted) due to resonance effects; the other (desired) due to the recycling connections.

In addition to electrical limitations, armature mass, bearing friction, air resistance, and other factors increase relay closing time. These, plus residual magnetism, increase opening time. The maximum cycle length (lowest frequency) is determined by the condenser leakage resistance.

Noninstantaniety of the recycling process suggests that (4) should be rewritten:

$$T_c = K + 2.303RC \log_{10} \frac{E}{RI_d}, \tag{6}$$

where: K = response constant, usually small, often negligible, determined experimentally after I_d is set.

As $RI_d = E_d$, where E_d is the drop-out voltage, (6) may be restated:

$$T_c = K + 2.303RC \log_{10} \frac{E}{E_d} \tag{7}$$

Salient Properties of the Parallel Circuit. If a charged condenser is connected across a resistor, current through the resistor at any time after the circuit is closed is shown by (2), the applied voltage in this

[3] Lanterman, W. F., in Henney, Keith, "Radio Engineers' Handbook," New York, 1941, 125–178: Pender, H., and Warren, S. R., Jr., "Electrical Circuits and Fields," New York, 1943, 108–133. Included bibliographies useful.

case being that to which the condenser was charged—in most instances the battery voltage.

By arranging the contacts of a relay-condenser parallel circuit so that the condenser is recharged whenever the relay current falls below the drop-out value, a parallel oscillator (Fig. 4) is obtained. Length of one cycle is indicated by (4), I being the current in the relay coil, E being some voltage between the pull-in voltage of the relay and the battery voltage; its exact value being determined by relay response and

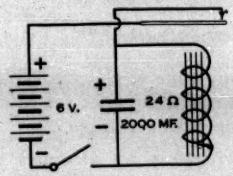


FIG. 4. Simplest form of the parallel oscillator.

external circuit characteristics. Possibility of unwanted resonant oscillation is indicated by (5), it being remembered that the relay has one inductance when the armature is up, and another when it is down.

Salient Properties of the Relay Holder. When a relay, in a circuit having "lumped" electrical components, is shunted by a condenser, the "closed" time of the relay contacts exceeds the "closed" time of the actuating circuit by the time necessary to partially discharge the condenser through the relay coil. In Fig. 5, assuming that all components

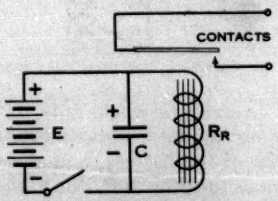


FIG. 5. Relay holder circuit.

are "pure," the duration of relay closure is shown by:

$$T_{cc} = A - K_c + 2.303 R_r C \log_{10} \frac{E}{R_r I_d}, \tag{8}$$

where:

T_{cc} = Time (seconds) that the relay contacts are closed,
A = Time (seconds) that the actuating circuit is closed,
R_r = Relay resistance (ohms),
K_c = Closing time of relay (in seconds, usually negligible).

This formula is only applicable when the condenser attains full charge during the time interval A. When the charge on the condenser is less than battery voltage, but more than E_d, dependable operation may be attained in some instances, but erratic operation is common.

Salient Properties of the Relay Accelerator. It has long been known by electrical workers that the closing of a relay can be accelerated by increasing the applied voltage. Theoretical increase in response is shown by:

$$\frac{T_2}{T_1} = \frac{E}{E_r}, \tag{9}$$

where:

T_2 = Operating time at elevated voltage (seconds),
T_1 = Normal operating time (at rated voltage),
E_r = Rated operating voltage.

In practice, this increase is not attained, due to relay and circuit limitations. Where only a slight increase in speed is desired, with relays having an ample margin of safety, this may be obtained by "straight" overvolting. Safe limit of this overvolting is set by the heat-dissipating ability of the relay under operating conditions.

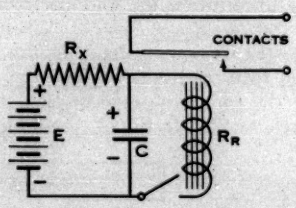


FIG. 6. Schematic diagram of relay accelerator.

Where "straight" overvolting would burn up the relay coil, momentary overvolting, by use of the circuit of Fig. 6, in which the initial applied voltage is that of the battery, and the steady state voltage is that of the battery minus the drop in R_s due to the relay holding current, will produce the desired acceleration. If a satisfactory holding current is midway between the pull-in and drop-out currents of the relay, the value of the series resistor, R_s, is shown by:

$$\begin{aligned} E - I_h R_s &= E_h, \\ E_h &= I_h R_r, \\ I_h &= \frac{I_p + I_d}{2}, \\ R_s &= \frac{E - E_h}{I_h}, \end{aligned} \tag{10}$$

where:

R_x = Series resistance (ohms),
I_h = Holding current (amperes),
E_h = Holding voltage (volts).

Determination of the optimum value of the shunt condenser by rigorous computations is a somewhat difficult matter. If the condenser is too small, the acceleration will not be the maximum possible for the voltage used; if the condenser is too large, the relay coil will heat unduly. In practice, the value of C, which is not critical, should be such that, when charged from the battery and immediately connected across the relay, the terminal voltage falls to the holding value during the pull-in time of the relay. C can be evaluated from a modification of (3):

$$T_p = 2.303RC \log_{10} \frac{E}{E_h},$$

$$C = \frac{T_p}{2.303R \log_{10} \frac{E}{E_h}}, \tag{11}$$

where: T_p = pull-in time of relay (seconds).

In practice, the nearest stock capacity between $.75C$ and $1.5C$ will be satisfactory.

With this type of relay accelerator, a finite time must elapse after the circuit is opened before the shunt condenser is recharged to a value permitting full accelerated operation. This time is shown by a modification of (3):

$$T_r = 2.303R_xC \log_{10} \frac{E}{E_h}, \tag{12}$$

where: T_r = Recharge time of condenser (seconds).

In all relay accelerator circuits, the relation:

$$E - I_hR_x = E_h, \tag{13}$$

must be adhered to. If the applied voltage (left hand term) is greater than the holding voltage, E_h, undue heating of the relay may result; while if the applied voltage is the lesser, the relay may open again after "slamming."

Where an inter-operation rest period is undesirable, the series resistor, R_x, may be shorted, when the relay is open, by means of an additional pair of relay contacts. This permits immediate condenser recharge.

Although accelerations exceeding five to one can be secured by means of condenser-resistor overvolting of a relay, this method should not be carried too far just because the formula indicates that it will work. It

is perfectly possible to destroy a relay coil completely by means of a condenser discharge.

Salient Features of the Relay Retarder. If, in the relay holder circuit (Fig. 5), a resistor is inserted in the positive battery lead ahead of the condenser, a relay retarder (Fig. 7) is produced. This circuit has two time delay effects. After circuit closure, the relay does not close until the potential across the relay reaches the pull-in voltage. When the circuit is opened, the relay does not open until the condenser discharge current through the coil falls to the drop-out value.

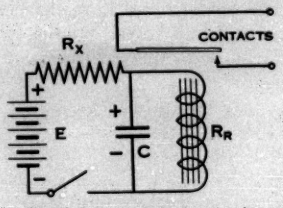


FIG. 7. Relay retarder circuit.

Assuming that the condenser is fully discharged, and that E_c is the voltage across the condenser, T_{cd}, the closing delay of the relay is shown by:

$$E_c = E - R_x(I_c + I_r), \tag{a}$$

$$I_r = \frac{E_c}{R_r}; \qquad E_c = I_r R_r, \tag{b}$$

$$I_r R_r = E - R_x(I_c + I_r), \tag{c}$$

$$I_r = \frac{E - R_x I_c}{R_r + R_x}, \tag{d}$$

$$I_c = \frac{E}{R_x}\epsilon^{-T/R_x C}, \tag{e}$$

$$I_r = \frac{E - E(\epsilon^{-(T/R_x C)})}{R_r + R_x}, \tag{f}$$

$$\frac{E - I_r(R_r + R_x)}{E} = \epsilon^{-T/R_x C}, \tag{g}$$

$$\log_\epsilon\left(\frac{E - I_r(R_r + R_x)}{E}\right) = -\frac{T}{R_x C}, \tag{h}$$

$$T = 2.303 R_x C[\log_{10} E - \log_{10}\{E - I_r(R_r + R_x)\}]. \tag{i}$$

$$T_{cd} = 2.303 R_x C[\log_{10} E - \log_{10}\{E - I_p(R_r + R_x)\}]. \tag{14}$$

The value of the series resistor necessary to produce a desired closing delay can be found by evaluating (14) for R_x, either by direct algebraic manipulation (somewhat difficult), or by insertion of trial values, after all other values are set.

Limit of the delay possible is indicated by:

$$\frac{E}{R_r + R_x} => I_p. \tag{15}$$

If the total resistance of the circuit is too high to permit passage of the pull-in current, the relay will not close.

When the circuit is opened (Fig. 7), the condenser discharges through the relay, which will not open until the current falls to the drop-out value. The opening delay, T_{od}, is shown by:

$$T_{od} = 2.303 R_r C \log_{10}\left[\frac{E - R_x I_r}{R_r I_d}\right], \tag{16}$$

provided the condenser, when the battery circuit is opened, has attained maximum charge.

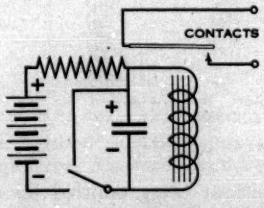


FIG. 8. Elimination of "holdover" in relay retarder.

In many practical applications, the holding effect of the relay retarder, and the mandatory "rest period" between operations, to permit the condenser to discharge fully, so that the next operation will also be retarded, are not desirable.

Use of a single-pole double-throw switch, connected so that the condenser is short-circuited (and hence discharged) when the battery circuit is opened (Fig. 8), eliminates both of these undesired conditions. The same effect can be produced by use of an auxiliary relay, which permits remote control over a two-wire line. A common battery circuit of this type is shown in Fig. 9. The auxiliary relay may be operated from any desired supply or control mechanism.

Several devices embodying the principles of the relay retarder, such as the Ward-Leonard series of condenser-delay relays, have been placed on the market in recent years.

Salient Features of the Impulse Limiter. If the recycling contacts of the series oscillator (Fig. 3) are disconnected, the relay will close once as soon as the circuit is closed, then will open again when the charging current of the condenser, flowing through the relay coil, falls to the drop-out value. Duration of this closure is shown by:

$$T_{cc} = 2.303 R_r C \log_{10} \frac{E}{R_r I_d}, \qquad (17)$$

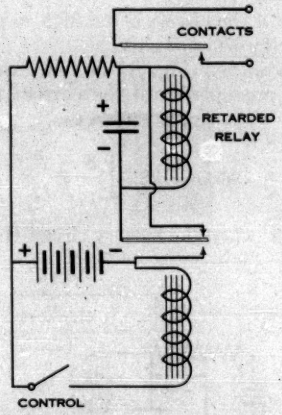


FIG. 9. Use of auxiliary relay to eliminate "Holdover" in relay retarder circuit.

the relay response constant being omitted for simplicity. This, the simplest form of the impulse limiter, is essentially a "one shot" device, the number of operations being limited to a few per hour, with electrolytic condensers, and a few per week, with paper condensers, due to the slow dissipation of the condenser charge.

To reduce the rest period between operations, a resistor may be shunted across the condenser, as in Fig. 10. The value of this resistor

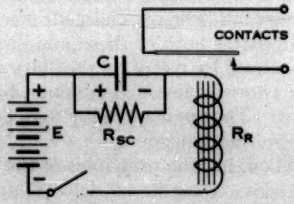


FIG. 10. Impulse limiter for relatively infrequent operation.

must be such that:

$$I_d > \frac{E}{R_r + R_{sc}}, \tag{18}$$

$$R_{sc} > \frac{E}{I_d} - R_r, \tag{19}$$

otherwise, the relay will remain closed after the condenser is charged, and there will be no limiter action. A correction for condenser leakage current is sometimes necessary.

The duration of the relay contact closure is best found by use of the following approximations. When the condenser is fully charged, and hence drawing no current, the voltage across it cannot exceed the battery voltage less the drop across R_r due to the circulating current.

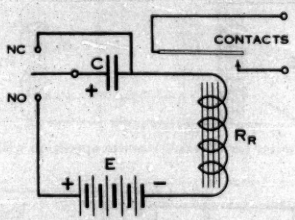


FIG. 11. Relay limiter for repeated operation without "rest period."

This equals the drop across R_{sc} from the same cause, and is shown by:

$$\frac{ER_{sc}}{R_{sc} + R_r}. \tag{20}$$

If (20) is now substituted for E in (17), the formula:

$$T_{cc} = 2.303 R_r C \log_{10} \frac{ER_{sc}}{R_r I_d (R_{sc} + R_r)}, \tag{21}$$

is obtained. This nonrigorous approximation is quite satisfactory in practice. When R_{sc} is large with respect to R_r, as is usual, (20) approaches E in value, and (21) approaches (17).

When the control switch (Fig. 10) is opened after one closure and opening of the relay contacts, the condenser is charged to a voltage indicated by (20). Theoretically, it would take an infinite time before this cycle could be repeated with the same holding time; actually, the required "rest period" may be quite short. If it is assumed that the cycle can be repeated when the current through the shunt resistor reaches a certain fraction, $1/N$, of the drop-out current of the relay, the duration

of the "rest period," in seconds, is shown by:

$$T_{rp} = 2.303 R_{sc} C \log_{10} \frac{EN}{I_d(R_r + R_{sc})}. \qquad (22)$$

When this "rest period" is undesirable, the condenser may be discharged by use of a double-throw control switch (Fig. 11). Duration of relay contact closure is shown by (17) provided the control circuit is closed for T_{cc} or longer. A snap type control switch should be used, so that the operator will not forget to discharge the condenser.

A dual relay, common battery, limiter, designed for remote operation over a wire line is shown in Fig. 12. The control relay, at the left,

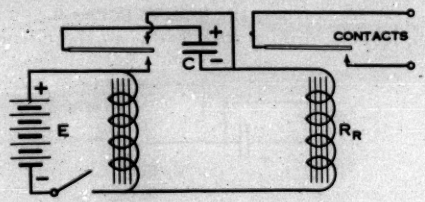


FIG. 12. Relay limiter for repeated remote operation without "rest period."

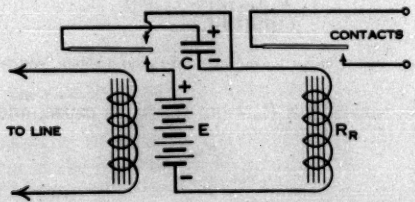


FIG. 13. Relay limiter with control circuit electrically independent of limiter circuit, and without "rest period."

replaces the switch of Fig. 11. A limiter in which the control relay is electrically independent of the limiter circuit is shown in Fig. 13.

A somewhat different system, based on the same general principles, is shown in Fig. 14. This limiter closes a circuit for a definite time after the control circuit has been opened. When the control circuit is closed, the condenser, C, is connected across the battery, and hence charged. Opening the control circuit releases the relay armature, which rises, disconnecting the condenser from the battery, and connecting it across the limiter (upper) relay. Limiter contact closure is shown by (17).

If the control (lower) relay of Fig. 14 is connected as a holder, the "carryover" limiter can be made to operate a definite time after the con-

trol circuit is opened, or only if the control circuit is open for a finite time, indicated by (8).

Salient Properties of the Impulse Doubler. If the "carryover" limiter of Fig. 14 is so modified that the condenser both charges and discharges

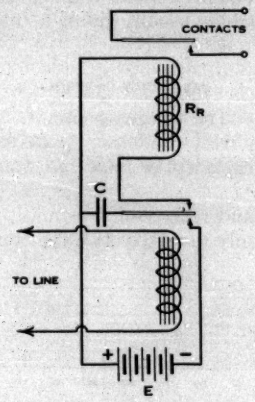


FIG. 14. "Carryover" limiter, to close the controlled circuit for a definite time after the control circuit has been opened.

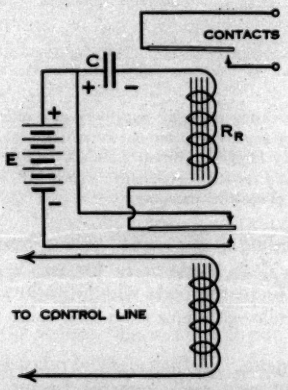


FIG. 15. Impulse doubler.

through the limiter relay (Fig. 15), then the limiter relay contacts will close once each time the control circuit is closed; and once each time it is subsequently opened. Contact closures will be of equal duration on the charge and discharge phases only if each phase starts from the same

base—with the condenser fully charged at the start of the discharge, and completely "empty" at the beginning of the charge, for example. Durations of closures are shown by (4) under these conditions.

If constants are so chosen that the charge and discharge phases are of equal duration, and equally spaced in time, the impulse doubler functions as a frequency doubler, usually giving a "messy" approximation of a square wave output.

FREQUENCY CONTROL.

General Discussion. The foregoing circuits and formulae show that the response rate of any relay-condenser device can be altered by changing the input voltage, capacity, or relay resistance; or by adjusting the relay sensitivity.

Voltage, capacity, and relay coil resistance, in most instances, are conveniently variable only in relatively large steps, permitting changes

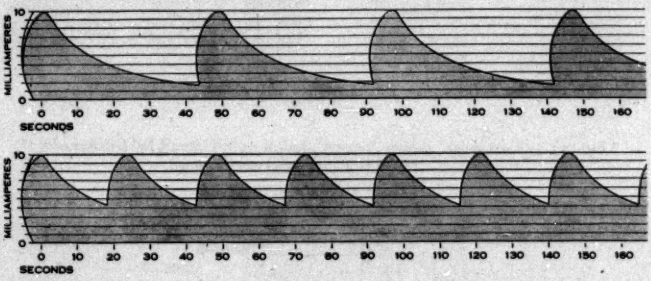


FIG. 16. Effect of decreasing the relay sensitivity of a series oscillator. Upper curve, relay adjusted for maximum sensitivity (drop-out current 1.6 ma.). Lower curve, relay adjusted for medium sensitivity (drop-out current 4.2 ma.). Relay resistance was 1,104 ohms, supply voltage (D.C.) 12, and capacity (nominally) 18,000 MF. in each case. Peaks of the curves are rounded because of recorder damping.

in rate usually exceeding 5 per cent., when commercial components are used.

Outlines of adjustment methods which produce substantially "stepless" rate changes in relay-condenser devices, using standard commercial components, follow.

Sensitivity Adjustments. Adjustment of relay sensitivity will change the operating time of any relay control circuit. Within reasonable limits, both the pull-in and drop-out currents of a relay vary directly as the spring tension.

Results of altering the sensitivity of the relay in a series oscillator are shown in Fig. 16. Maximum sensitivity is shown in the upper curve; medium in the lower. These changes were obtained by relay spring

adjustments. Substantially identical changes are produced by use of magnetic shunts.

Although these adjustment methods can be made to work within rather wide limits, commercial relays usually work *dependably* through a rather narrow range of spring tensions and field strengths, so that the utility of sensitivity adjustments is somewhat limited.

Frequency Control by Use of Supplementary Resistors. Failure of other methods to supply a simple, convenient, and "stepless" rate control system led to adoption of supplementary resistors, which are connected to alter the effective resistance of the relay coil.

The frequency of a relay oscillator can be altered by use of either a series or a parallel control resistor. Although it is possible to use both

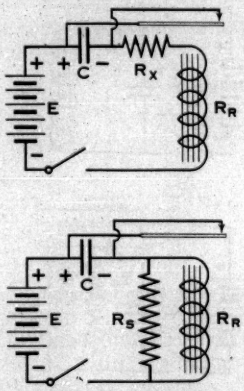


FIG. 17. Control of frequency of a series oscillator. Upper diagram shows series resistor; lower diagram shows shunt resistor.

types in one oscillator, this is unnecessary in practice, leading to undesirable circuit complexities, and "interlock" of controls.

With series resistor control of a series oscillator (Fig. 17, upper diagram), the control resistor, R_s, can have any value from zero to $\frac{E}{I_p} - R_r$. With greater values, the relay will not close, and hence the oscillator will not function. Increasing the series resistance lengthens the cycle duration, formula values being found by substituting $R + R_s$ for R in (4).

Shunt control (Fig. 17, lower diagram) is slightly harder to evaluate. In (4) substitute $\frac{R_s \cdot R_r}{R_r + R_s}$ (from Kirchoff's law for parallel resistors) for

R, giving formula:

$$T_c = \frac{R_s}{R_r + R_s} R_r C \, 2.303 \log_{10}\left[\frac{E}{I_d}\left(\frac{R_r + R_s}{R_r \cdot R_s}\right)\right]. \qquad (23)$$

As R_s increases, (23) approaches (4), becoming identical when R_s is infinite. Within reasonable limits, frequency increases with decrease in R_s, the limiting value being where duration of one cycle approximates response time of the relay.

Control of the frequency of the parallel oscillator by series or shunt resistors is accomplished similarly. Frequency with series control (upper diagram, Fig. 18) is found from (4) by substituting $R_r - R_s$ for R. Control limitations are the same as with the series oscillator.

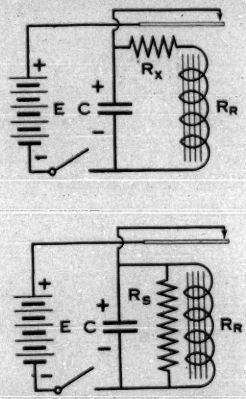


FIG. 18. Frequency control with parallel oscillator. Upper diagram—series connection; lower diagram—shunt connection.

Shunt control of a parallel oscillator (lower diagram, Fig. 18) is accomplished similarly to that in a series oscillator. Frequency can be computed from (23), it being assumed that the condenser terminal voltage, at full charge, is the battery voltage.

Timing of the relay holder can be controlled by either series or shunt resistors, the connections being obvious. Series resistor controlled time is found by substituting $R_s + R_r$ for R_r in (8). If $\frac{E}{R_s + R_r}$ is less than I_p, the holder will not work. Timing of the shunt controlled circuit is shown by:

$$T_{cc} = A - K_c + \frac{R_s}{R_r + R_s} R_r C \, 2.303 \log_{10}\left[\frac{E}{I_d}\left(\frac{R_r + R_s}{R_r \cdot R_s}\right)\right], \qquad (24)$$

which is derived from (8) and (23).

The relay accelerator may be retimed by use of a resistor in series with the relay only, as in Fig. 19. This raises the effective operating voltage of the relay, and hence reduces the acceleration with a given applied voltage. Time change due to this resistor is shown by:

$$\frac{T_2}{T_1} = \frac{E}{I_p(R_r + R_{x2})}, \tag{25}$$

where R_{x2} is the value of the series control resistor (ohms).

Timing of the impulse limiter can be altered by use of either series of shunt resistors; or by use of a "bleeder" across the condenser (21). Series-controlled time is found from (17), using $R_x + R_r$ in place of R_r, and bearing in mind that $\frac{E}{R_r + R_x}$ must exceed I_p, or the limiter relay will not close. When a control resistor is shunted across the relay coil, "formula" time is shown by (23).

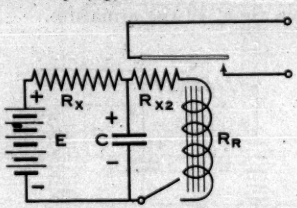


FIG. 19. Series resistance control of the relay accelerator.

Time control of the carryover limiter is by use of the same series and shunt circuits as the "direct" limiters, and formula times are computed in the same manner with the same formulae.

CYCLING.

As the operating cycle of a simple relay oscillator usually consists of a relatively long "armature down" interval, followed by a very short "armature up" interval, the duration of which is usually constant, regardless of frequency, and determined by relay response characteristics, some modification of the contacting cycle is desirable in many practical applications.

Obvious and well-known methods, such as the insertion of a surge filter in the controlled circuit, use of dashpots, air dampers, or spring pendulums, will not be described. A few of the more dependable electro-mechanical methods of cycle control will be outlined; many more undoubtedly await discovery.

SIMPLE CYCLE CONTROL.

Series Oscillator. No simple method of modifying the operating cycle of the series oscillator is known.

Shunt Oscillator. Contacting cycle of the shunt oscillator may be modified by insertion of a series resistor in the supply lead, which produces, in actuality, an oscillating relay retarder (Fig. 20).

If the condenser is fully discharged at the moment the circuit is closed, the time lapse until the first pull-down of the armature is shown by:

$$T_{cd} = 2.303R_xC[\log_{10} E - \log_{10}\{E - I_p(R_r + R_x)\}]. \qquad (14)$$

(Derivation of this formula is given under "relay retarder.")

The armature will now remain down until the condenser discharge current falls to the drop-out value. The "down time" of the armature is shown by:

$$T_d = 2.303R_rC \log_{10} \frac{I_p}{I_d}, \qquad (26)$$

where T_d is the "down time" of the armature.

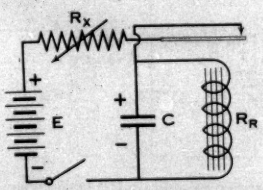


FIG. 20. Parallel or shunt relay oscillator with resistor to control "armature up" portion of cycle.

When the relay current reaches the drop-out value, the armature rises, and the condenser recharges through R_x. Recharge from I_d to I_p (*not* from zero to I_p) is shown by:

$$T_r = 2.303R_xC \log_{10}\left[\frac{E - I_d(R_r + R_x)}{E - I_p(R_r + R_x)}\right], \qquad (27)$$

where T_r is recharge time (seconds).

Succeeding cycles, after the first half cycle, are composed of 26 plus 27, or:

$$T_c = 2.303C\left[R_x \log_{10}\frac{E - I_d(R_r + R_x)}{E - I_p(R_r + R_x)} + R_r \log_{10}\frac{I_p}{I_d}\right]. \qquad (28)$$

And successive half cycles, after the first, will be of equal length when $T_d = T_r$, or:

$$R_x\left[\log_{10}\frac{E - I_d(R_r + R_x)}{E - I_p(R_r + R_x)}\right] = R_r \log_{10}\frac{I_p}{I_d}. \qquad (29)$$

It will be noted, with this circuit, that each change of contact cycle produced by varying R_x is accompanied by a change in frequency. Additional variations in frequency can be obtained by use of the series and shunt resistors already described, or by a resistor in series with the condenser only.

Push-Pull Series Oscillator. Inflexibility of the simple series oscillator led to experiments from which a push-pull series oscillator (Fig. 21) was developed. Once started, it is quite dependable, but a special starting switch, which energizes one side a fraction of a second before the other, is necessary, or the device will suffer from "electrical indecision."

Length of each half cycle of a push-pull series oscillator is shown by (4), length of a full cycle being $2T_c$.

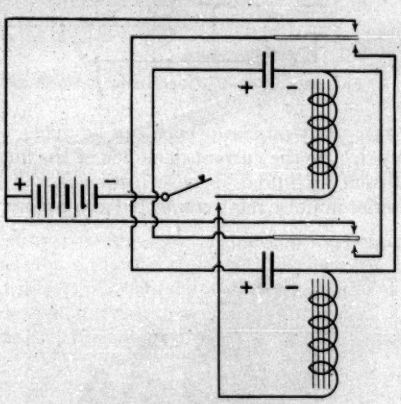


FIG. 21. Push-pull series oscillator.

Use of double-pole double-throw contacts on the relays will permit coupling to various devices. Use of one set of contacts on each relay in an obvious circuit to produce a square wave generator is to be avoided, because of a short circuit each half cycle. A push-pull oscillator, however, can be used to produce two square wave outputs, either in phase or 180° out of phase, by use of two three-pole double-throw relays.

Push-Pull Parallel Oscillator. Although two shunt oscillators can be connected in a push-pull arrangement, operation is very tricky, special relay contacts are required, and the device needs an inordinate amount of servicing. Duration of one-half cycle is shown by (4), of a full cycle by $2T_c$ (4).

Series-shunt Oscillator. An interesting oscillator, embodying some features of both the series and shunt types, has been developed by Dr. S. W. Grinnell, of Stanford University (Fig. 22). Experiments with

this oscillator indicate that the shunt condenser should have about 1/10th the capacity of the series condenser, and that the relay-shunt condenser circuit is resonant. Although very stable at certain frequencies, frequency changes are quite difficult.

Impulse Relay Oscillators. By addition of a pair of armature-controlled contacts, any impulse ("ratchet") relay can be used in a relay

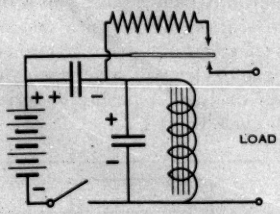


FIG. 22. Series-shunt oscillator of Grinnell.

oscillator (Fig. 23). Any oscillator type can be made, and any type of control can be used. As the current demands of the impulse type relay are great, the capacity required, in most instances, will be enormous.

Circuit of a series impulse relay connected as a square wave generator comprises Fig. 24.

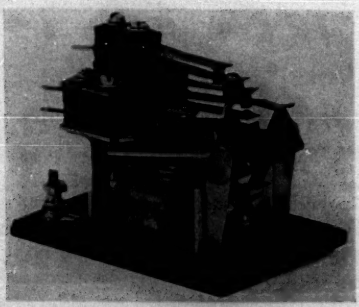


FIG. 23. Contact arrangement for oscillating impulse relay.

The impulse relay oscillator is very stable and dependable, in operation, but very noisy, and prone to "kick itself to pieces" after a few hours of operation, due to the relatively large mass of the armature.

COMPOUND CYCLE CONTROL.

Introduction. Although several minor variations of the fundamental relay oscillator circuits will give some control of the contacting cycle,

it is often necessary to control relatively large amounts of power by means of a relay oscillator.

Under the general heading "compound cycle control" will be outlined a few of the simpler methods of improving the contacting cycle by use of auxiliary relays.

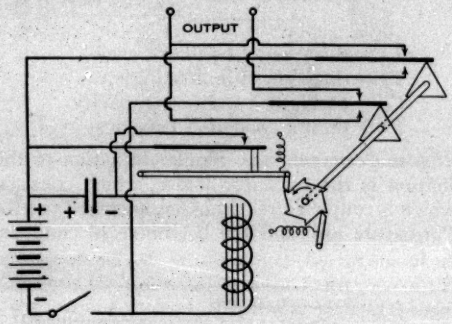


FIG. 24. Series impulse relay oscillator connected as a square wave generator. The capacity required here is very large.

Oscillator-Limiter Combinations. Combinations of a relay oscillator with a limiter having a shorter closure time than one oscillator cycle, form a very satisfactory control unit. The limiter contacts can be closed for any desired fraction of the oscillator cycle. Combination of a series oscillator and limiter is shown in Fig. 25. A parallel oscillator

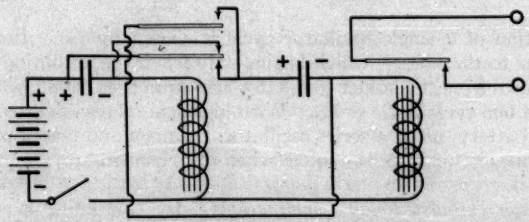


FIG. 25. Oscillator-limiter combination.

can be similarly connected, but is slightly more subject to contact troubles.

Oscillator frequency is shown by (4); "closed" time of the limiter by (17). When the oscillator and limiter relays are of identical characteristics, and operate from the same power supply, ratio of oscillator cycle

duration to limiter closure time is determined by the ratios of the respective capacities (30).

$$\frac{T_0}{T_l} = \frac{2.303 R_0 C_0 \log_{10} \frac{E}{RI_d}}{2.303 R C_l \log_{10} \frac{E}{RI_d}} = \frac{C_0}{C_l}, \qquad (30)$$

in which:

T_0 = duration of oscillator cycle,
T_l = duration of limiter cycle,
C_0 = oscillator condenser capacity,
C_l = limiter condenser capacity.

Oscillator-Holder Combinations. Similar in action to the oscillator-limiter combination is the oscillator-holder, which consists of a relay oscillator, to which is coupled a relay holder, arranged to produce a time lag between "armature up" and the beginning of condenser recharge (Fig. 26).

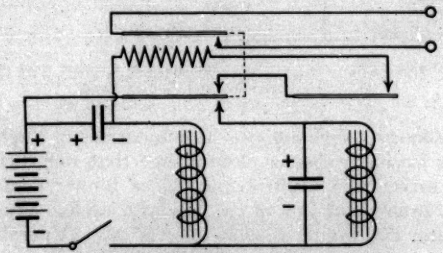


FIG. 26. Oscillator-holder combination.

Duration of a single oscillator cycle is shown by (4). Inter-cycle delay, due to the holder, which begins only when the actuating voltage is withdrawn from the holder (oscillator armature rises), is shown by (8). Length of one cycle is $T_c + T_{cc}$. With identical relays operated from a common battery, using a series oscillator, the open and closed phases in the output circuit will be equal when the condensers are of equal capacity.

Oscillator-Impulse Relay Combinations. By combining a relay oscillator with an impulse relay, a device which is substantially a submultiple generator is secured. Fundamental frequency is set by the constants of the oscillator; submultiples are determined by the range and connections of the impulse relay contacts. One form of this combination is shown in Fig. 27. A more practical arrangement, requiring, however an enormous condenser, is to connect the impulse relay as a limiter. Constructional cost is reduced, in most instances, by cascading a relay oscillator, relay limiter, and impulse relay.

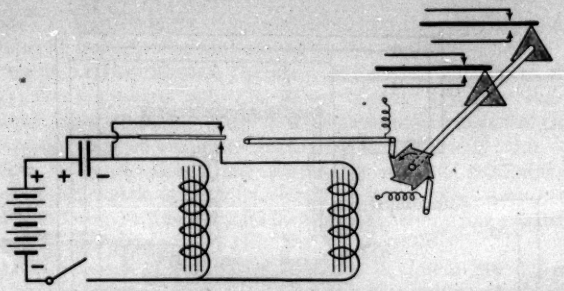


FIG. 27. Relay oscillator-impulse relay combination.

CONTACT PROTECTION.

In many of the relay-condenser devices here described, the condenser, at some stage of operation, is discharged by means of a "dead short" introduced by the relay contacts. As the resistance of the drain circuit is very low, the current through the drain contacts is momentarily very high, and usually far in excess of the rated capacity. This leads to welding of the contacts, sticking, oxidation, and typical "cone and crater" erosion.

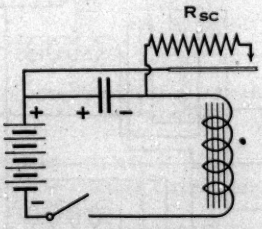


FIG. 28. Contact protecting resistor R_{sc} inserted in drain circuit of series oscillator.

This trouble may be corrected, in most instances, by connection of a small resistor, valued at about one-half ohm per battery volt, in series with the drain circuit (Fig. 28).

Although this protective resistor theoretically increases the time necessary to discharge the condenser, its small value makes the effect negligible in most instances.

PRACTICAL APPLICATIONS.

Introduction. Laboratory and commercial uses for a low frequency oscillator of great simplicity and dependability are numerous. In many instances, the relay oscillator satisfactorily fills the gap between the

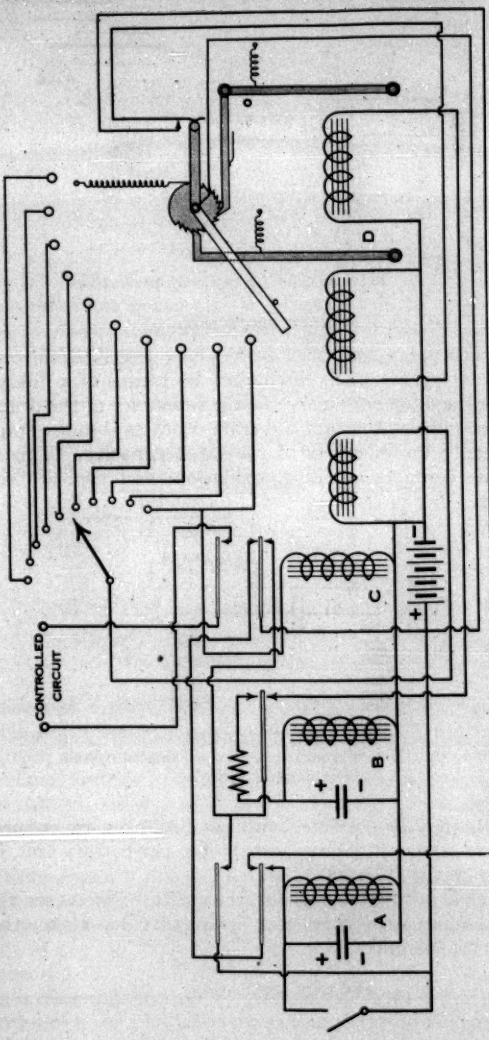


FIG. 29. Operations timer employing relay oscillator, stepping switch, and auxiliaries. Many variations of this general system are possible.

lower limit of dependability of standard vacuum tube oscillators and the upper limit of most mechanical switching devices.

For use in field equipment, the relay oscillator is superior to all other types in that it requires only one power supply, and may share that supply with other devices. A few of the simpler applications of the relay oscillator and its close electrical relatives will be outlined here.

Timers. The simplest timer employing a relay oscillator is the oscillator itself, which acts as an electrical metronome. An oscillator coupled with a stepping switch and locking relay comprises a satisfactory and versatile operations timer (Fig. 29).

When the switch of this timer (Fig. 29) is closed, holding relay *A* closes, starting the oscillator, *B*. When the first half cycle of operation (not dependable as to duration) is completed, and the oscillator armature pulls down, the stepping switch, *D*, is energized, and the contact arm moves to the first point, energizing the latching relay, *C*, which pulls down and locks. This starts the process which is being timed,

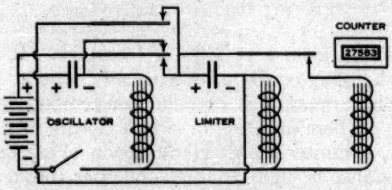


FIG. 30. Electric stopwatch consisting of an oscillator, a limiter, and an electric counter.

deenergizes the holding relay, *A*, which, after a short delay due to the shunt condenser, opens the holding circuit, and connects the release circuit to its control mechanism.

When the arm of the stepping switch has progressed to the preset release point, the release coil of the latching relay, *C*, is energized. This allows the stepping switch arm to return to "home" position, where it opens the "off normal" contacts, and hence the release circuit. The timer is now ready for the next operating sequence.

Use of a second or third bank of stepping switch contacts permits control of a number of simultaneous operations. Connection of a "time out" control to any contact position allows a long operation to be completed without further stepping.

An electric stopwatch, for determining the length of time intervals, can be made by mounting a pair of contacts on a standard electric counter, such as the Veeder-Root type UD, and connecting it as an oscillator. Because of the large current demands of the counter coil, a condenser of stupendous dimensions is required.

A less costly, weighty, and bulky interval timer consists of a relay oscillator, limiter, and counter, in cascade (Fig. 30). This is not subject to serious contact troubles, is substantially immune to all but the most severe vibrations, and is economical of battery power.

To produce audible signals of one-half second duration at five second intervals, the timer of Fig. 31 was constructed. This, designed for use

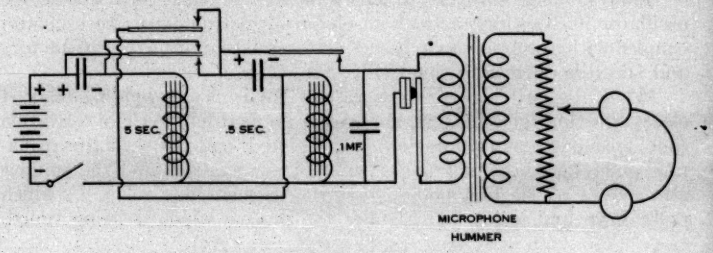


FIG. 31. Timer giving audible signals.

in aircraft, operated from a $7\frac{1}{2}$ volt "C" battery. It is for devices of this type that the relay oscillator is especially suited.

For timing short operations, combinations embodying a relay holder are convenient. Where duration of switch closure is a potential source of timing error, a limiter-holder combination (Fig. 32) may be used. Where line resistance is high, a large condenser (broken lines, Fig. 32)

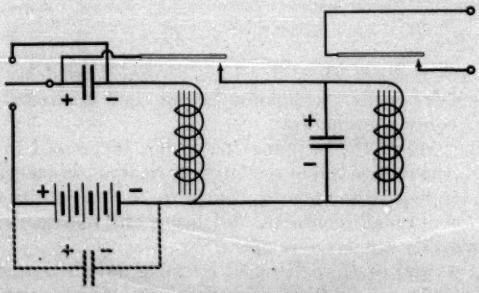


FIG. 32. Limiter-holder combination for short interval process timing.

s shunted across the supply line close to the control device. This permits use of smaller line wires, and prevents limiter-holder surges from influencing other equipment sharing the battery and line.

Stepping Devices. Although widely used in communications, stepping switches and their close electrical relatives have found little favor in noncommunication work because of their reputed uncontrollability.

The major problem with continuous rotation stepping switches is to return the wipers to "home" position after completion of a sequence of steps.

Continuous rotation stepping switches can be homed rapidly by use of the conventional self-homing connection, in which one contact bank

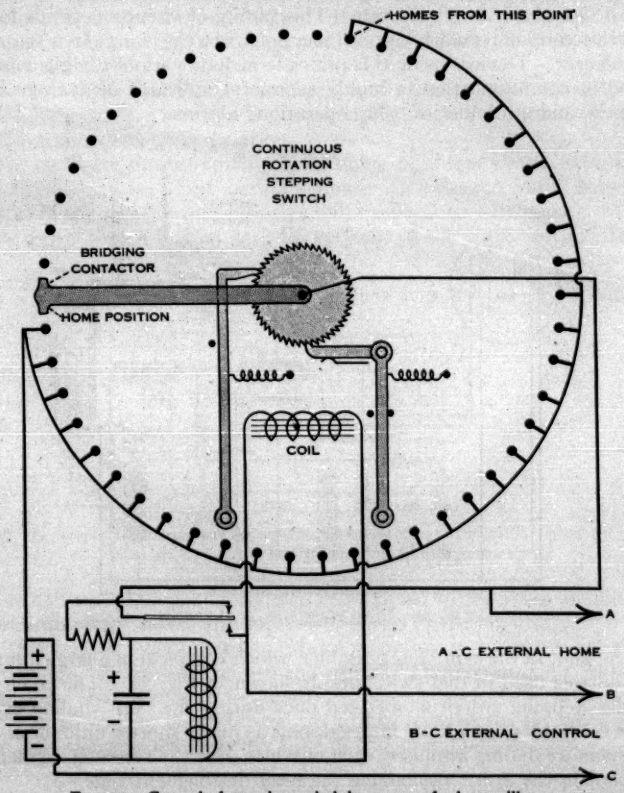


FIG. 33. Control of stepping switch by means of relay oscillator.

is so connected that as soon as one point is reached, the next is energized, Stepping may be slowed by connecting a *very large* condenser across the stepping coil.

When relatively slow stepping is desired, the stepping circuit can be driven by a relay oscillator (Fig. 33). The usual "time out" circuits

may be used at any contact. Where the duration of the stepping impulse is critical, an oscillator-limiter driving combination is desirable.

Direct limiter connection of a stepping switch is theoretically possible, but the limiter condenser capacity required is of the order of 0.1 *Farad*. With a separate limiter, dependable operation can be secured with a capacity not exceeding 0.001 Farad (1,000 MF.).

"*Shot in the Arm*" *Circuits*. This family of circuits depends for its operation upon regularly applied increments to the charge in a reservoir condenser. Devices using this principle include various dialing controls used in communication, a highly successful indicator of average wind speeds, and a number of "skip operation" alarms.

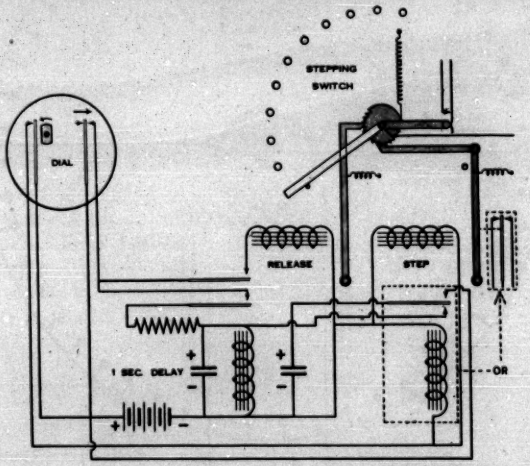


FIG. 34. "Shot in the arm" reset disabler.

A "shot in the arm" reset disabler, which is typical of a large number of controls used in dial switching, is shown in Fig. 34. The reset coil of the stepping switch is operated once during the "up" dialing cycle; then is disabled for a finite interval, plus as many shorter finite intervals as there are dialing impulses, each of which give the reservoir condenser a "shot in the arm," prolonging the reset disablement time.

Similar in its action is the average wind speed indicator developed by the U. S. Weather Bureau.[4] Recently this general principle has been applied, with slight modifications, to a successful recording anemometer.[5]

[4] Middleton, W. E. K., "Meteorological Instruments," Toronto, 1942, 129.

[5] Operational data and constants in Grinnell, S. W., "Some Instruments Used by Division 10, NDRC, at Dugway Proving Ground for the Continuous Recording of Micrometeorological Conditions," OSRD 6088, 1945, 6–8, 24–25, Fig. 3. Complete technical description of this instrument, by Dr. Grinnell, is forthcoming.

For giving an alarm whenever a mechanism omits one of a series of equally-spaced operations, a skip-operation alarm, consisting of a relay holder with the holding time slightly greater than one operation interval, but less than two intervals, is useful. Each operation charges the holder condenser fully. The condenser discharges continually through the relay. If operations occur regularly, recharge occurs before the relay current falls to the drop-out value. If an operation is skipped, relay current falls below drop-out value, and the relay armature rises, actuating the alarm.

Square Wave Generators. Connection of an impulse relay oscillator as a square wave generator is shown in Fig. 24. Additional obvious combinations need no description.

An oscillator-limiter combination which produces waves with each half cycle "square," but permits almost any desired ratio between lengths of the positive and negative half cycles at any frequency within its range, is shown in Fig. 35. Wave form of the entire output, or of

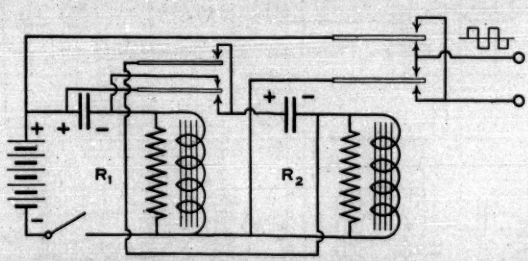


FIG. 35. Oscillator-limiter combination used as a square wave generator. Frequency is controlled by R_1, ratios of lengths of successive half cycles by R_2.

either half cycle independently, can be modified by insertion of suitable "corner breaker" filters at appropriate places in the circuit.

"Keep Alive" Mechanisms. Many recording instruments are sluggish, due to friction between pen and paper. This is commonly rectified by tapping the instrument frequently. Introduction of alternating electrical impulses into the indicator circuit not only serves the same purpose, but shortens the response time of the instrument markedly.

Numerous successful "Keep Alives" or "Electric Diddlers" have been made. One model, developed by Dr. S. W. Grinnell, of Stanford University, consists of a parallel relay oscillator with a special secondary winding on the relay coil. This is connected in series with the instrument movement.

Another similar device (Fig. 36) uses stock parts, and allows considerable regulation of "kick" amplitude. This type is shunted across the instrument movement through two small condensers.

Neither of these "Keep Alives" give, or are intended to give, sinusoidal currents. All that is necessary is that the "kicks" in either direction be of the same amplitude and duration, so that the instrument pen position is not biased by the "Keep Alive."

The Grinnell-Volman Pump. Combination of a relay oscillator and a solenoid-actuated diaphragm pump (Fig. 37) by Drs. S. W. Grinnell

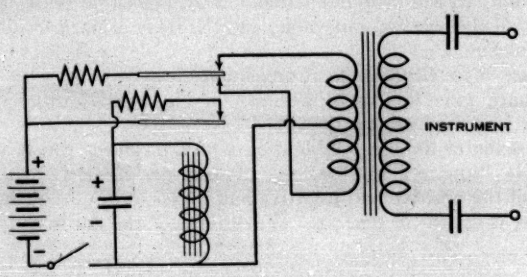


FIG. 36. "Keep Alive" using standard parts.

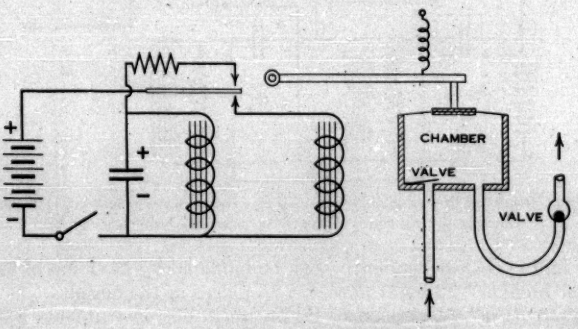


FIG. 37. One form of the Grinnell-Volman pump.

and D. H. Volman produced a family of small pumps of great controllability and efficiency. Numerous variations, such as the use of a series oscillator and a limiter-connected solenoid, are possible. A push-pull pump of this type, with one solenoid actuated by the charging current of the condenser, and the other by the discharge current, offers interesting possibilities.[6]

Alarms and Annunciators. Applications of the relay oscillator, holder, and limiter to alarm and annunciator systems are numerous, and

[6] A complete laboratory report covering the operation and application of this type of pump is in preparation by Drs. Grinnell and Volman.

most of them obvious. By cascading condenser-relay circuits, a wide variety of signals can be produced, but extensive cascading is not always satisfactory, as trouble increases roughly as the square of the number of circuit components.

Typical of annunciators employing relay-condenser circuits is the alarm of Fig. 38, which gives a thirty second intermittent buzz when the line is degenerized (danger) and a thirty second continuous buzz when it is reenergized (all clear).

In operation, when the line is deenergized, the armature of relay *A* (Fig. 38) rises. This changes the panel light from green to red, and

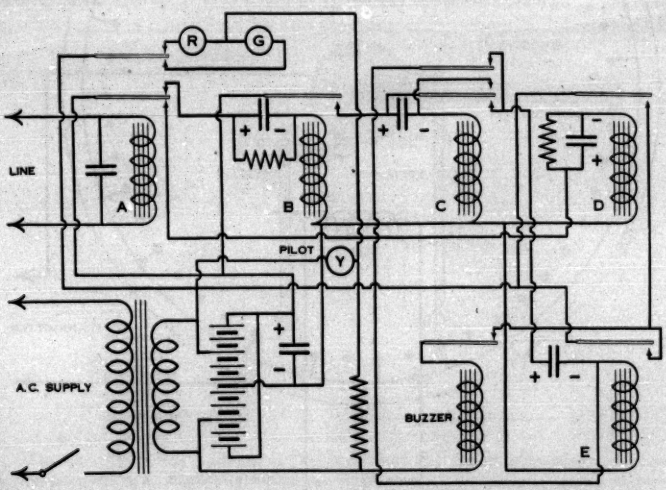


FIG. 38. Alarm circuit producing an intermittent buzz for 30 seconds when the line is deenergized, and a 30 second continuous buzz when the line is reenergized.

energizes limiter circuit *B*, which has a holding time of 30 seconds. This starts the relay oscillator *C*, which has a period of one second. This in turn operates the limiter *D*, which has a contact closure of one-half second. The limiter operates a buzzer, giving an audible signal of one-half second buzzes spaced one-half second apart. After 30 seconds, the armature of *B* rises, and no more signals are emitted until the situation changes.

When the line is reenergized, the armature of line relay *A* pulls down, changing the panel light from red to green, and energizing limiter *D*, which has a closure of 30 seconds. This actuates the buzzer, producing a continuous buzz for 30 seconds, after which there are no more signals until the situation changes.

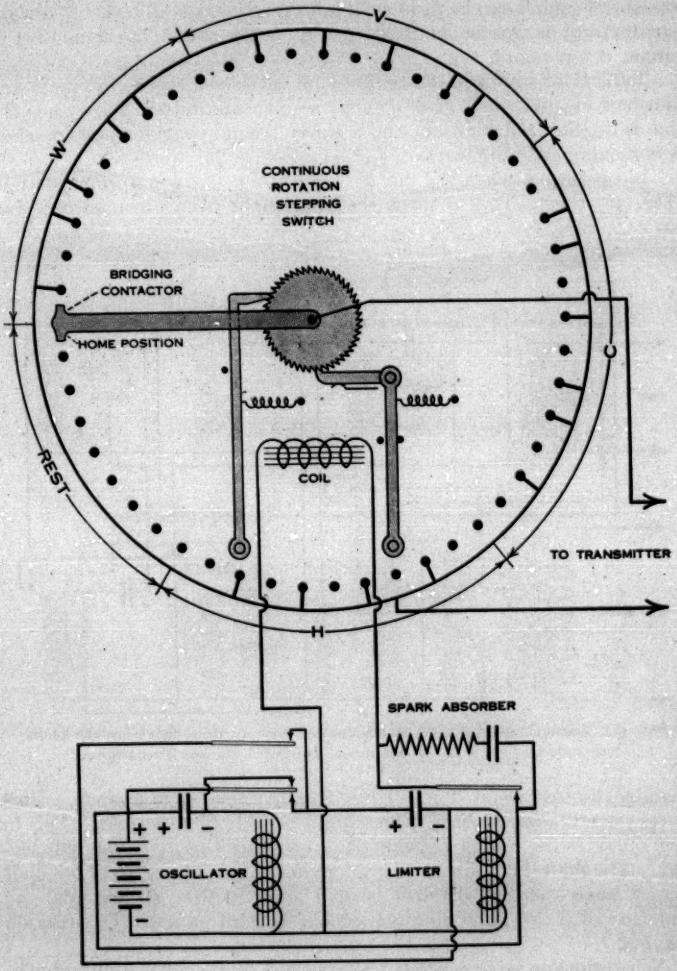


FIG. 39. Automatic transmitter.

Both limiters (*B* and *D*) are equipped with bleeders, to permit re-operation on about 60 second headway. The series oscillator-limiter combination (*C* and *E*) was used in preference to a parallel oscillator of similar temporal characteristics because of greater dependability.

Automatic Signaling Devices. Combination of a relay oscillator, limiter, and stepping switch (Fig. 39) led to development of a light, compact, efficient, and dependable automatic transmitter which had definite advantages over the more conventional motor-driven code wheels and tape senders.

Contacts of the stepping switch are wired to transmit the call W V C H in Morse, and homing is at the same speed as sending. Transmission speed is controlled by the stepping rate, which is determined by

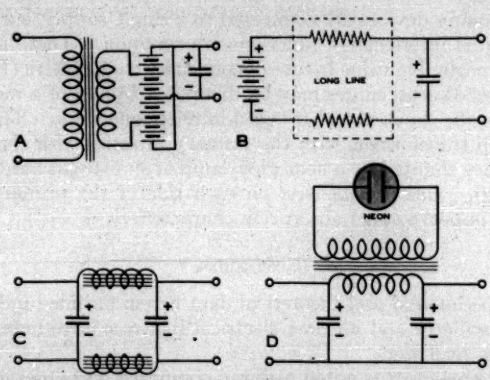


FIG. 40. Power supply adjuncts. *A*—rectifier and filter for operation of relay-condenser devices from the A. C. supply. *B*—condenser shunted across long supply line to compensate for line resistance and inductance. *C*—surge filter for supply line. *D*—surge absorber.

the period of the oscillator. The limiter closure can be the same for all transmission speeds within the range of the transmitter.

Practical considerations limit the usefulness of oscillator-stepping switch transmitters to about 200 elements, despite the theoretical possibility of employing a stepping switch having ten banks of 50 contacts each. When longer messages must be transmitted automatically, the amount of auxiliary equipment required becomes excessive, and some other transmission method, such as a tape sender, is more economical.

Power Supplies. Direct current relays, such as are used in most relay-condenser devices, are commonly operated from batteries. Where lines are short, and neither the line nor the battery are shared with other equipment, direct connections are the rule.

Where line length is relatively great, and where line or battery are shared with other equipment, the resistance and inductance of the line may introduce trouble because most relay-condenser devices draw very heavy currents for a small part of each operating cycle. By connecting a large condenser across the supply line, as close to the controlled device as possible, much or all of this trouble may be eliminated (Fig. 40, *B*). The large condenser acts as an electrical surge tank.

Operation of relay-condenser devices from the A. C. supply line, by use of a conventional transformer-rectifier-filter combination (Fig. 40, *A*) is entirely practicable. A 1,000 MF. condenser is usually adequate. Regulation may be improved by use of a bleeder across the output. Grounding of the transformer core and some point in the D. C. circuit is important.

Where many devices are connected to a single supply, surges, kicks, and unwanted interlocks of functions are common. These may be reduced to a negligible value by use of condenser-choke filters (Fig. 40, *C*).

In extreme cases, surges may be dissipated by use of a modification of the "acoustic shock absorber" used in telephone work. This consists of a step-up transformer, with the primary in series with the line, and the secondary shunted by a neon glow lamp or similar gas discharge tube (Fig. 40, *D*). Small condensers on each side of the primary winding sometimes improve surge absorption characteristics.

CONCLUSIONS.

The experimental and theoretical data herein outlined indicate that the relay oscillator and its close electrical relatives have many applications in electrical work.

A close agreement is noted between computed and observed characteristics of these devices, even though the computations, by disregarding relay inductance, are nonrigorous.

Relay-condenser devices have a useful range of from more than 100 cycles per second [7] to about 10 cycles per hour. This range overlaps the useful range of vacuum tube oscillators at the higher end, and that of mechanical switching devices at the low frequencies.

Although relay-condenser devices can be cascaded to perform complex sequences of operations, their maximum utility is in the realm of relatively simple devices, particularly when weight and power supply limitations must be considered. Extreme ruggedness and easy repairability of most commercial relays make relay-condenser devices ideal for use in hazardous locations.

[7] Recent developments indicate that these ranges will be markedly extended in the near future. The new Stevens-Arnold "Millisec" relay, for example, will operate dependably in .001 sec.

ACKNOWLEDGMENTS.

In the preparation of this paper, many suggestions, ideas, and helpful criticisms were received from numerous sources. This cooperation is gratefully acknowledged.

To Mr. Philip R. Nourse, of the Douglas Aircraft Co., Santa Monica, Calif., the writer is indebted for many helpful suggestions concerning the application of relay oscillators to industrial control problems.

Dr. S. W. Grinnell, of Stanford University, assigned to war researches collateral to the author's, greatly expedited the experimental work by field testing a number of oscillators. His suggestions, criticisms, and discussions of oscillator problems have been of great assistance in the preparation of this paper.

To Tec. 3, Evelyn L. Kroona, WAC, the writer is indebted for diligent and competent stenographic assistance.

"Rocket Radio" For Moon to Earth Broadcasts Foreseen.—The technical possibility of building a compact "rocket radio" weighing less than 100 pounds and capable of transmitting signals from the moon to the earth was advanced by Westinghouse scientists commenting on the proposed moon-bound rockets the Army expects to construct within 18 months.

Referring to recent rocket forecasts of the Army Air Force guided missile branch, Dr. J. A. Hutcheson, associate director of the Westinghouse Research Laboratories, declared it now is possible to build a small radio set of sufficient power to send its signal 240,000 miles from the moon to the earth.

"A 100-watt transmitter capable of beaming ultrashort waves from the moon to the earth would weigh less than 50 pounds," he said. "Its power supply, consisting of several batteries the size of automobile storage batteries, would require only an additional 50 pounds thanks to wartime improvements in battery construction. The whole set could be enclosed in a cabinet about three feet in length and one foot in depth and width."

Dr. Hutcheson pointed out that the set would have to be hermetically sealed in order to operate in the near vacuum of the ionosphere and from the surface of the moon itself. At an altitude of 60 miles above the earth where the ionosphere begins air pressure approaches only one-billionth that of normal atmosphere.

"Assuming a rocket speed of 4,000 miles per hour," Dr. Hutcheson continued, "it would require about 60 hours to reach the moon. If the rocket radio were operating all this time it would apply too heavy a drain on the batteries. Therefore, we could use an electric clock mechanism to turn the batteries on for one minute out of each hour. This would give sufficient battery life to provide hourly broadcasts while the rocket ascended and for several days on the moon itself.

"Such hourly broadcasts not only would report the progress and landing of the rocket but also might be used to record temperature changes on the moon," he said. "Since resistance of wire changes according to the temperature, this change could be made to vary the frequency modulation of the radio transmitter and change the tone of the signal which would be received on earth.

"Station M-O-O-N might also help us discover many facts which now are largely a matter of conjecture. It generally is assumed that there is no moisture on the moon because we can see no evidence of atmosphere in the form of clouds and haze. Instruments aboard the rocket could check on this fact and relay the answer to earth by radio."

The problem of landing a 4,000-mile-an-hour rocket on the moon without smashing delicate radio equipment was explained by the Westinghouse scientist.

"The proximity fuse developed during the war holds the key to providing a 'soft' landing for moon-rockets," he declared. "When the rocket approaches the moon, a tiny radio set weighing only a few ounces would detect the fact. This automatically would turn on reverse rockets and turn off the forward rocket power. Such tiny radio outfits in anti-aircraft shells produced during the latter part of the war detected the proximity of airplanes and exploded the shell at the right moment. By starting reverse rocket power at the right moment in this application, these proximity radio sets would make possible rocket landings without damaging delicate instruments."

R. H. O.

APPLICATION OF CIRCUIT THEORY TO THE DESIGN OF SERVOMECHANISMS.*

BY

ALBERT C. HALL, Sc. D.†

SUMMARY.

Circuit analysis methods have proved to be powerful tools in the design of automatic control systems. A servomechanism and its principal components are defined, and the reason for employing this type of automatic control system pointed out. The general requirements of stability, accuracy, and speed of response are emphasized and methods are explained by which they may be calculated for particular systems. The similarity between a servomechanism and a feedback amplifier is pointed out.

The types of circuits employed in correcting the dynamical and steady state responses of servomechanisms are illustrated and methods of choosing circuit parameters are developed.

I. INTRODUCTION.

It is frequently necessary to control the position of a device or the state of a process in accordance with indications or signals supplied by a controlling instrument. If the power required to operate the device or process is large compared with the power available from the controlling instrument, power-amplifying means must be provided. If the amplification of these controlling signals is carried out by a machine or automatic device (rather than by a human operator), the complete system is known as an automatic control system and the element that amplifies the controlling signals and operates the device or process is known as an automatic controller.

Automatic control systems can be divided into two types, depending upon the source of the signals that actuate the controller. If the signals supplied to the controller are determined not only by the controlling instrument but also by the position or state of the device or process being controlled, the system is known as a closed-cycle control system. However, if the signals supplied to the controller are not a function of the state of the quantity being controlled the system is known as an open-cycle control system.

The term "servomechanism" is used to denote a particular class of closed-cycle automatic control system. If the controlling signals to the system change frequently and rapidly and the primary problem is to cause the controlled device to "follow" these signals, the system is called

* This paper is adapted from a thesis, "The Analysis and Synthesis of Linear Servomechanisms" submitted to the department of Electrical Engineering, M.I.T., in May, 1943, published by The Technology Press, and distributed as a restricted publication by Division 7, N.D.R.C.

† Massachusetts Institute of Technology.

a servomechanism. If the controlling signals do not change, or change relatively slowly, and the primary problem is to prevent extraneous causes from affecting the function controlled by the system, the system is called a regulator. For example, servomechanisms are used to steer ships, to control airplanes, to automatically tune radio receivers, to position guns, in numerous telemetering applications, and in general in those automatic control applications where "following" accuracy is of prime importance. Regulators, for example, are used to control temperature, to maintain a constant liquid level, etc.

The distinction between a servomechanism and an open-cycle control system is made most easily by comparing an example of each. Suppose a telemetering problem is encountered in which it is necessary to repeat or reproduce at a higher power level the indication of an instrument. One solution to the problem is illustrated by Fig. 1. The motion of the master instrument is measured and translated into an alternating voltage by a pair of magnetically coupled coils. This alternating voltage is

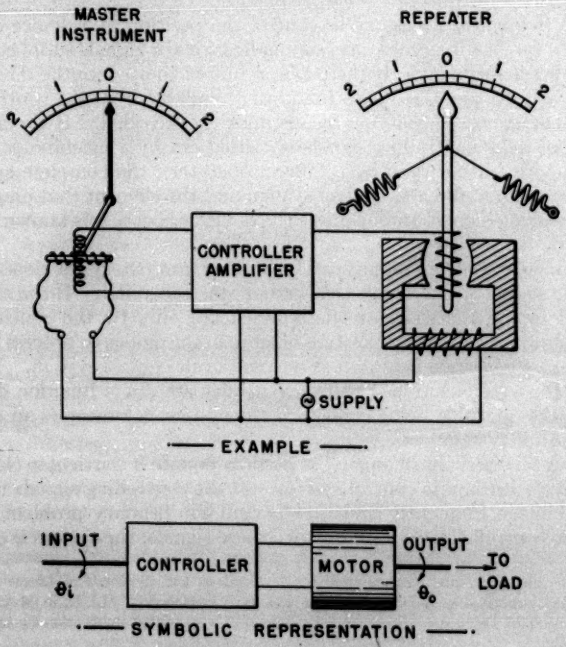


FIG. 1. Example of an open-cycle control system.

amplified and supplied to a torque motor, the rotor of which is coupled to the movement of the repeater instrument. If the voltage from the measuring coils of the master instrument is proportional to the deflection of its movement, and the voltage output of the amplifier is made proportional to its input, the torque developed by the torque motor will be proportional to the deflection of the master instrument. The deflection of the repeater will, therefore, be proportional to the deflection of the master instrument provided the springs restraining the movement of the repeater are properly designed.

The automatic control system described above is an open-cycle control system, since the signal supplied to the amplifier (or controller) is a function of (or generated by) the motion of the master instrument only. This system has many obvious disadvantages and should be avoided when high accuracy is demanded. In order that the deflection of the repeater be an accurate measure of the deflection of the master instrument, the transfer characteristics of every element in the system must remain constant. Thus the amplifier gain must be constant, the supply voltage must not change, the restraining springs must retain their calibration, etc.

The same problem can be handled by the closed-cycle control system illustrated by Fig. 2. The deflection of the master instrument is measured by a pair of coils similar to those employed in the open-cycle controller previously described, except that both coils are rotatable, one by the movement of the master instrument and the other by a shaft coupled to the repeater. The signal voltage from the measuring coil is amplified and used to actuate the driving motor. The motor is coupled to the movement of the repeater and to the reference coil. When the master instrument is displaced, the motor is actuated and revolves through an angle sufficient to re-establish the measuring coils in perpendicular positions.

The system illustrated by Fig. 2 is a closed-cycle system since the signal actuating the controller is a function not only of the position of the master instrument but also of the position of the repeater. The advantage of a closed-cycle system over the open-cycle system is made clear by comparing the systems of Figs. 1 and 2. The accuracy of the closed-cycle system depends only upon the accuracy of the gearing from the repeater to the reference coil and the accuracy of the measuring coils, provided the controller amplifier and motor are sufficiently sensitive. The accuracy is relatively independent of amplifier gain, motor sensitivity, and supply voltage. However, the closed-cycle control system may be made unstable by improper design because of the feedback link from the motor output to the measuring coil.

Commonly used symbolic representations of open- and closed-cycle control systems also are shown in Figs. 1 and 2.

The following terms have been applied to denote the different portions of the servomechanism. The quantities defined are well established and while they have not been standardized, they are understood by those active in the field.

Input. The input is the controlling quantity. It may be the displacement of a shaft, a voltage or current, a temperature level, or a required liquid level. The deflection of the indicating meter in Fig. 2 is

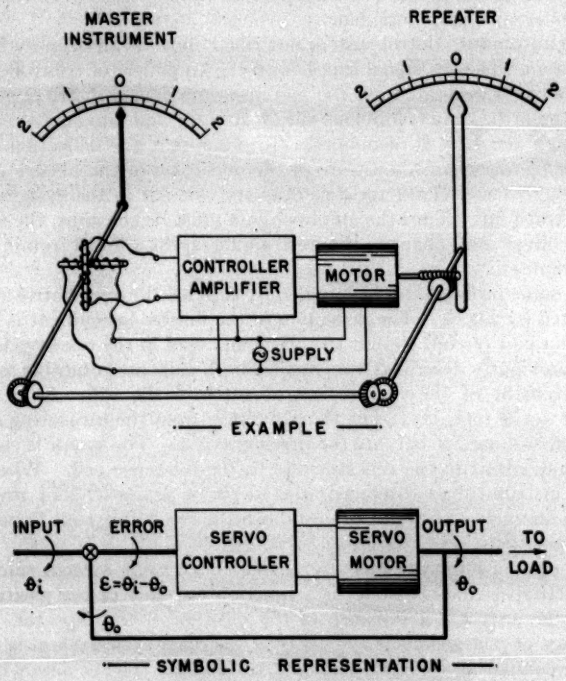


FIG. 2. Example of a closed-cycle control system.

the *input* to the automatic control system. If the input ordinarily does not change in value the system may be known as a regulator. The input is generally denoted as $\theta_i(t)$ as a time function and $\Theta_i(j\omega)$ as a function of frequency.

Output. The servo output is the position or state of the controlled quantity. It may be any of the quantities listed under the input. In the example of Fig. 2, the servo *output* is the deflection of the repeater

instrument. The output is represented as $\theta_0(t)$ as a function of time and $\Theta_0(j\omega)$ as a function of frequency.

Error. The servo error is the difference between the servomechanism input and the output. In the example of Fig. 2 the *error* is the angle by which the two mutually-coupled coils depart from being perpendicular. The error is commonly denoted as $\epsilon(t)$ as a function of time and $E(j\omega)$ as a function of frequency.

Error Detector. The error detector is the device which compares the input with the servomechanism output. The mutually coupled coils comprise the *error detector* in the system depicted by Fig. 2.

Servo Controller. The servo controller is the device whose input is the error and whose output controls the servo motor. In Fig. 2 the servo controller might be an a-c amplifier.

Servo Motor. The servo motor is the prime mover of the servomechanism. It is actuated by the servo controller and controls the servo load or output.

Servo Load. The servo load is the quantity upon which the servo motor operates. In Fig. 2 the servo load is the repeater movement and associated gearing.

II. SERVOMECHANISM RESPONSE CHARACTERISTICS.

An ideal servomechanism would be an automatic control device which would maintain perfect correspondence between the output and the input. Employing the terminology listed above it is a system that always maintains zero error no matter what is the nature of the input motion. Obviously an ideal servomechanism cannot be realized and the best that can be done is to so design the servomechanism that the correspondence which it maintains between the output and input is sufficient for the practical purpose for which it is intended; i.e., the error is sufficiently small for the application.

It is the problem of the servomechanism designer to determine in advance how well a proposed system will meet the requirements of a particular problem. In order to obtain an exact assessment of a system, the designer must prescribe mathematically the nature of the input which the servomechanism must follow, as well as the response characteristics of the servomechanism itself. Unfortunately, the input rarely can be represented in a mathematically precise fashion. However, the designer generally can predict the approximate nature of the input and if it is principally transient in character, study the response of the proposed servomechanism design to test inputs of similar nature to the actual input to which the system is subjected.

For example, a servomechanism is frequently required to follow a number of abrupt changes in input value, occurring in more or less random fashion. The engineer can handle this situation by determining the response of the proposed design to a "step function" (or instan-

taneous change in magnitude) of the input and making sure that the error in the system does not exceed the allowable limit and that the error is substantially zero before the next change in input is likely to occur. If a reasonable factor of safety is employed such an approach provides a sound engineering answer to the problem. It is common, therefore, to characterize the response of a servomechanism by prescribing the nature of its response to a step function.

The response of the servo to an input of mathematically expressible form is determined by writing the differential equations of the system or by employing operational calculus or Laplace transform theory.[1] Laplace transform theory is the most flexible tool and is used in this paper. In general the relation between the output and the input can be written as follows:

$$\Theta_0(s) = \frac{P(s)}{Q(s)}\,\Theta_i(s). \tag{1}$$

The function $\Theta_0(s)$ is the Laplacian transform of the output, $\Theta_i(s)$ is the transform of the input, and the rational function $\frac{P(s)}{Q(s)}$ is the characteristic of the particular servomechanism being studied. In most cases the function $\frac{P(s)}{Q(s)}$ can be found by writing the differential equation relating the output to the input and replacing the operator $\frac{d^n}{dt^n}$ by s^n.

After the characteristic $\frac{P(s)}{Q(s)}$ has been determined, the time response of the system to a particular input is found by a straight-forward though generally laborious procedure. The Laplacian transform of the input is first determined. Thus if the input is a step function of unit change in magnitude at zero time, $\theta_i(s)$ is equal to $\frac{1}{s}$. This is written mathematically as follows:

$$\begin{aligned} &\text{If} && \theta_i(t) = 0 && \text{when} && t < 0 \\ &&& \theta_i(t) = 1 && \text{when} && t > 0. \end{aligned} \tag{2}$$

Then

$$\Theta_i(s) = \mathfrak{L}[\theta_i(t)] = \frac{1}{s}. \tag{3}$$

The symbol $\mathfrak{L}[\ ]$ stands for "Laplacian transform of."

The Laplacian transform of other types of input function may be derived or determined from tables.[2]

The function $\Theta_0(s)$ is then written as a partial fraction expansion in

[1] Gardner and Barnes, "Transients in Linear Systems," John Wiley and Sons, N.Y.C., 1942.
[2] Gardner and Barnes, ref. cit.

order to express it as a sum of simple terms the inverse transforms of which are easily recognized.

$$\Theta_0(s) = \frac{P(s)}{Q(s)}\,\theta_i(s) = \frac{A_1}{s + a_1} + \frac{A_2}{s + a_2} + \cdots. \tag{4}$$

Finally the inverse transform of $\Theta_0(s)$ is found and the time function of $\theta_0(t)$ is determined. Thus

$$\theta_0(t) = \mathfrak{L}^{-1}[\Theta_0(s)] = \mathfrak{L}^{-1}\left[\frac{A_1}{s + a_1}\right] + \mathfrak{L}^{-1}\left[\frac{A_2}{s + a_2}\right] + \cdots, \tag{5}$$

in which $\mathfrak{L}^{-1}[\ ]$ stands for "the inverse transform of." Since

$$\mathfrak{L}^{-1}\left[\frac{1}{s + a_1}\right] = e^{-a_1 t}, \tag{6}$$

the time function $\theta_0(t)$ can be written

$$\theta_0(t) = A_1 e^{-a_1 t} + A_2 e^{-a_2 t} + \cdots. \tag{7}$$

If the system is stable all the exponents, a_1, a_2, etc., are positive or have positive real parts. The magnitude of the exponents determines the speed with which the servo output approaches a new position called for by the servo input.

The laborious portion of this procedure is centered in the determination of A_1, A_2, $\cdots$ and a_1, a_2, $\cdots$. The latter are found by solving the equation

$$Q(s) = 0, \tag{8}$$

since

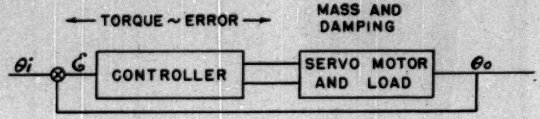
$$(s + a_1)(s + a_2) \cdots = Q(s). \tag{9}$$

The determination of the roots of Equation (7) is lengthy except in the most simple cases. In addition, the terms A_1, A_2, etc., are determined from rather involved expressions. However, once the transient response $\theta_0(t)$ is known it is generally possible to make accurate predictions of the nature of the system performance in particular applications. Examples of various possible transient responses of a simple system are illustrated in Fig. 3.

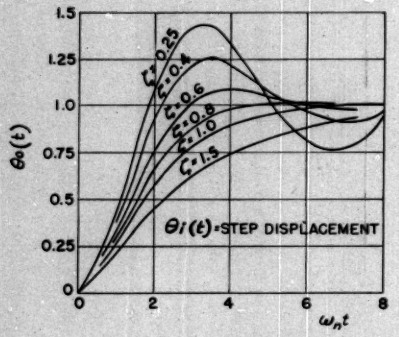
A more thorough treatment of the determination of the transient solution to automatic control problems is given in a paper by Brown and Hall.[3]

A second method of analyzing the response of a servomechanism is to allow the input, $\theta_i(t)$, to be a sinusoidal function of time and then to determine the steady-state amplitude and phase relationships between the output and the input over the important frequency range. This method, known as the sinusoidal or frequency response method, is

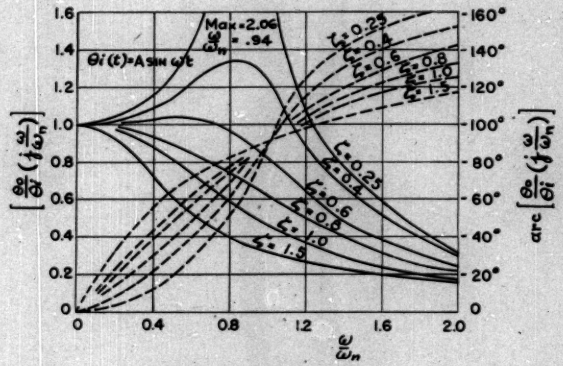
[3] Brown and Hall, Trans A. S. M. E., July, 1946.



SYMBOLIC REPRESENTATION



TRANSIENT RESPONSE



SINUSOIDAL RESPONSE

FIG. 3. Transient and sinusoidal responses of a second-order system.

identical with that employed in most circuit-analysis problems. The steady-state vector relationship between the output and the input when the input is a sinusoidal function of time is written

$$\frac{\Theta_0(j\omega)}{\Theta_i(j\omega)} = \frac{P(j\omega)}{Q(j\omega)} = B(j\omega)e^{j\phi(j\omega)}. \qquad (10)$$

The term "ω" is the angular frequency and j is equal to $\sqrt{-1}$. A knowledge of the amplitude function $B(j\omega)$ and the phase function $\phi(j\omega)$ completely determines the system response. Amplitude and phase responses of a simple system are illustrated in Fig. 3. The function $\frac{P(j\omega)}{Q(j\omega)}$ may be found by straight-forward circuit analysis methods or by replacing the Laplacian transform "s" by "$j\omega$" in Equation (1).

The frequency response of an ideal system would have a flat amplitude response and a zero phase response over the entire frequency range. Obviously this cannot be attained. The efforts of the servo designer are applied toward making the response characteristics of a proposed system approach as close as necessary to the ideal.

The sinusoidal amplitude response of a system indicates a great deal about the corresponding transient response. Peaks in the amplitude response occur at about the same frequencies at which the transient response has natural frequencies, and the damping of these natural frequencies is measured by the magnitudes of the peaks in the amplitude response. The correspondence between the transient and frequency response of a simple system is illustrated by Fig. 3.

A general design rule based on the system frequency response is summarized as follows. If the peaks in the amplitude response are limited to values of approximately one and one-third to one and one-half, and if the predominant peak (generally the first) occurs at a frequency at least three times the inverse of the period allowed for a transient to disappear, the system should be satisfactory for the application.

For example, if a system is required to respond to sudden inputs with such a rapidity that the error must be not more than a few per cent. of its initial value within one second, the lowest natural frequency of the system should be at least three cycles per second. The designer can then feel confident that the system will meet its requirements.

Thus a knowledge of the amplitude and phase response of a proposed system is of great value to the designer. This information can be supplied as curves such as are shown in Fig. 3, as analytical expressions, or both. One way in which the characteristics can be represented is depicted in Fig. 4. Here the vector nature of the relationship between the output and the input is emphasized and the locus swept out by the tip of the vector ratio of the output to the input is determined over the frequency range of interest. The frequency parameter is noted at

points along the locus. Thus at a particular frequency the amplitude response is given by the length of the vector from the origin to that point, and the phase response is the angle between that vector and the reference axis. The curves of Fig. 4 were drawn for the same case shown

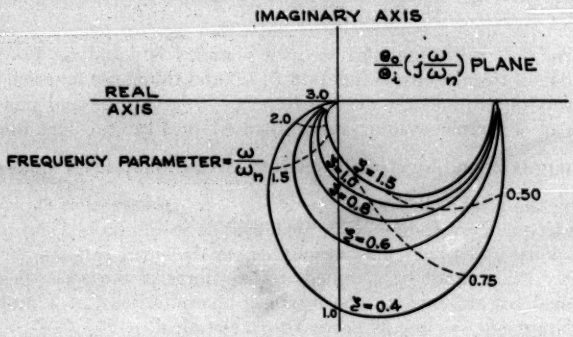


FIG. 4. Complex plane representation of the sinusoidal response of a second-order system.

in Fig. 3 and one set of curves presents the same information provided by the two sets of curves giving the sinusoidal response in Fig. 3.

III. SERVOMECHANISM TRANSFER FUNCTION.

A third and most useful method of presenting the frequency response data is developed as follows. In most servomechanisms the feedback element from the output to the input and the error measuring element are of such a nature that their transfer function may be considered to be independent of frequency. (See the block diagram of Fig. 2.) Under this circumstance the performance of the entire system is determined once the relation between the output and the error is written

$$\frac{\Theta_0}{E}(j\omega) = KG(j\omega); \tag{11}$$

this relation may be combined with the equation defining the error

$$E(j\omega) = \Theta_i(j\omega) - \Theta_0(j\omega) \tag{12}$$

and the frequency response of the system determined.

$$\frac{\Theta_0}{\Theta_i}(j\omega) = \frac{KG(j\omega)}{1 + KG(j\omega)}. \tag{13}$$

The only variable in the above equation is the function, $KG(j\omega)$. This function has been termed the transfer function of the servomechanism. It is a product of two terms: first, a factor that is independent of fre-

quency, represented by "K;" and second, a factor dependent upon frequency represented by $G(j\omega)$.

The function $KG(j\omega)$ also may be represented by a curve or locus in the complex plane, and such a curve is most useful to the designer. Fig. 5 presents the loci of a typical system. The magnitude of $KG(j\omega)$ for a particular frequency is given by the length of a vector from the origin to the point on the locus corresponding to that frequency; the phase of $KG(j\omega)$ for the same frequency is given by the angle that the same vector makes with the reference axis. The servomechanism frequency response is found by a simple graphical means from the locus of the transfer function, $KG(j\omega)$. For example, Equation (13) indicates

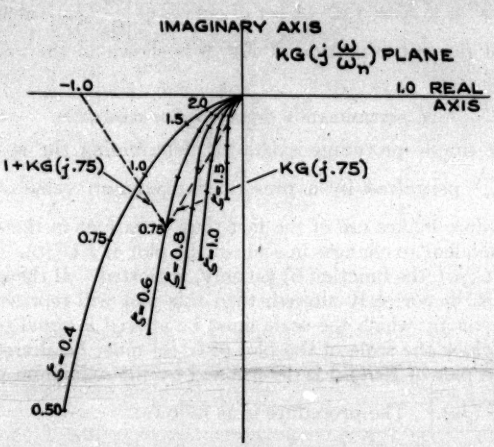


FIG. 5. Transfer-function loci of a second-order system.

that at a particular frequency, 0.75, the function $\frac{\Theta_0}{\Theta_i}(j.75)$ is the ratio of two vectors, $KG(j.75)$, represented by a vector from the origin to the point 0.75 in Fig. 5 and $1 + KG(j.75)$, represented by the vector from the point $(-1 + j0)$ to the point 0.75. The amplitude response of the system is the ratio of the magnitudes of these vectors, and the phase response is the angle included between these vectors.

The general nature of the amplitude response may be obtained also by drawing in the complex plane curves of constant $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$. These curves are circles and are shown in the upper part of Fig. 6. If a servo could have a transfer function locus that lay along one of these circles,

the system amplitude response would be independent of frequency and equal to the value of M for which the circle was drawn. The center and radius of each circle is dependent upon M and the relation is given in Fig. 6. The frequency at which the transfer function locus crosses one of these circles is the frequency at which the magnitude of $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$ is equal to the value of M for which that circle is drawn. If the locus is tangent to a circle it indicates that a maximum or minimum in $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$ occurs at that frequency. Thus the amplitude response of the system whose transfer function is plotted in Fig. 6 has a peak of 1.5 at a frequency $\omega = 2$, if $K = 1.0$. A set of curves of $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$ calculated from the plot of the upper portion of Fig. 6 is shown in the lower part of that figure.

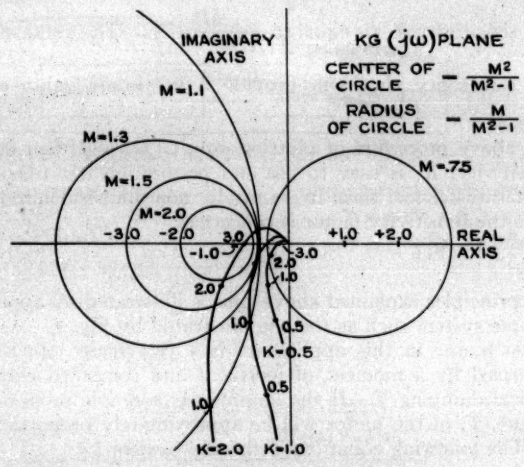
IV. DETERMINATION OF THE SYSTEM SENSITIVITY.

A very simple procedure exists for determining the system sensitivity, "K," permitted by a prescribed maximum value of $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$. This procedure makes use of the fact that variations in the sensitivity, K, are equivalent to changes in scale of the plot of $KG(j\omega)$. Instead of plotting $KG(j\omega)$, the function $G(j\omega)$ only, is plotted. If the scale of this plot of $G(j\omega)$ is correctly altered, then this plot will represent $KG(j\omega)$ and the factor by which the scale must be altered is equal to K. The factor by which the scale of the plot of $G(j\omega)$ must be altered to transform it to a plot of $KG(j\omega)$ is determined by the maximum permissible value of $\left|\frac{\theta_0}{\theta_i}(j\omega)\right|$. The procedure is as follows:

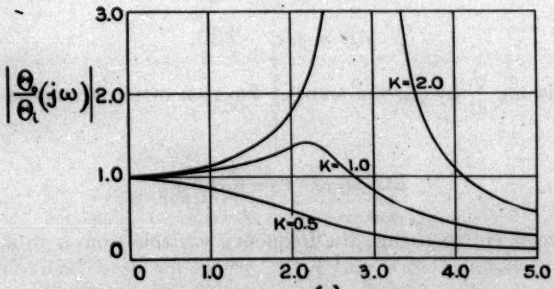
It has been shown that if the plot $KG(j\omega)$ is tangent to a circle whose center is $\frac{M^2}{M^2-1}$ on the negative real axis, and whose radius is $\frac{M}{M^2-1}$, that the function $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$ corresponding to this $KG(j\omega)$ will have a maximum value of M. Now a circle whose center is at $\frac{M^2}{M^2-1}$ on the negative real axis and whose radius is $\frac{M}{M^2-1}$ will have an intercept on the real axis equal to $\frac{M}{M+1}$, and the following ratio will be maintained;

$$\frac{\text{Center of circle}}{\text{Intercept on real axis}} = \frac{M}{M-1}. \tag{14}$$

The scale factor, K, can be readily found if a circle can be located on the plot of $G(j\omega)$ that is tangent to the locus, $G(j\omega)$, and whose center on the negative real axis is $\frac{M}{M-1}$ times its real-axis intercept. The location of such a circle can be found by a cut-and-try process with a



DETERMINATION OF SERVO SENSITIVITY



EFFECT OF SENSITIVITY ON SERVO RESPONSE

FIG. 6. Determination of the effect of sensitivity on the sinusoidal response of a servomechanism.

pair of dividers and is the work of but a few moments. Suppose such a circle has a center at A on the negative real axis of the $G(j\omega)$ plane. But it has been shown that if the scale were correctly chosen in order that the locus be a plot of $KG(j\omega)$, the center of the circle would be at $\frac{M^2}{M^2-1}$. Therefore the scale must be changed by the factor $\frac{M^2}{A(M^2-1)}$ and the sensitivity K_M is equal to $\frac{M^2}{A(M^2-1)}$. The value of K_M is the system sensitivity that will provide a maximum value of $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$ equal to M.

The above procedure of plotting only $G(j\omega)$ and then determining the sensitivity, K, is easy to use and permits greater freedom in the study of transfer loci since it essentially non-dimensionalizes the plots as far as the sensitivity factor is concerned.

Example:

The principles explained above can be illustrated by applying them to a simple system such as the one illustrated by Fig. 2. Assume that the servo motor in this application is a two-phase induction motor characterized by a moment of inertia J and combined electrical and mechanical damping, f. If the amplifier is a simple proportional one, the torque, T, of the motor will be approximately proportional to the error. The following equations define the system.

$$T = k\epsilon(t), \tag{15}$$

$$T = \frac{Jd^2\theta_0}{dt^2} + f\frac{d\theta_0}{dt}, \tag{16}$$

$$\epsilon(t) = \theta_i(t) - \theta_0(t). \tag{17}$$

Replacing $\frac{d}{dt}$ by $j\omega$, the transfer function of the system is readily found.

$$KG(j\omega) = \frac{k}{J(j\omega)^2 + f(j\omega)}. \tag{18}$$

It is convenient to change the frequency variable from ω to u, where $u = \frac{J\omega}{f}$:

$$KG(ju) = \frac{Jk}{f^2}\frac{1}{ju(1+ju)}. \tag{19}$$

In Equation (19)

$$G(ju) = \frac{1}{ju(1 + ju)}, \tag{20}$$

$$K = \frac{Jk}{f^2}. \tag{21}$$

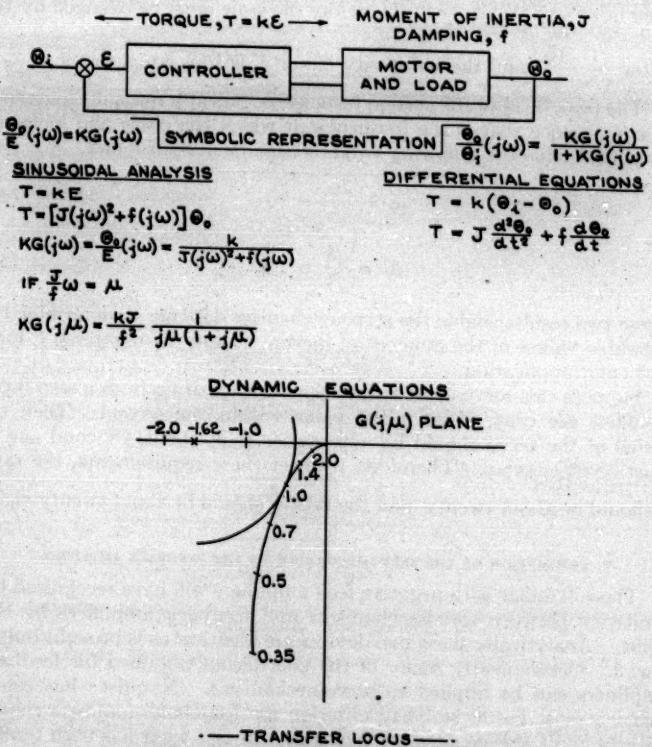


FIG. 7. Design summary of a second-order system.

A plot of $G(ju)$ is shown in Fig. 7 together with a summary of the analysis. Suppose the maximum value of $\left|\frac{\Theta_0}{\Theta_i}(j\omega)\right|$ is to be limited to one and one-third by proper choice of the sensitivity factor, K. The first step is to locate a circle corresponding to $M = 1\frac{1}{3}$ that is tangent to

the locus. If properly drawn the center of the circle will lie on the negative real axis and the ratio of its center to its intercept on the real axis will be four (in accordance with Equation (14)). The circle is readily located by a cut-and-try process, and its center is found to lie at 1.62.

If the scale of the graph is correctly altered, the center of this circle must lie at $\frac{M^2}{M^2-1} = \frac{16}{7}$. Therefore the scale must be changed by the factor $\frac{1}{1.62}\frac{16}{7}$, and the sensitivity factor K is thus equal to 1.4.

The peak of $1\frac{1}{3}$ in the system response occurs at a resonant frequency of $u = 1$, since that is the frequency at which the circle and locus are tangent. Thus the following two relations have been found.

$$\mu_r = \frac{J\omega_r}{f} = 1.0, \tag{22}$$

$$K = \frac{Jk}{f^2} = 1.4. \tag{23}$$

These two results enable the servomechanism designer to determine the allowable values of the moment of inertia, J, and the damping, f, for a particular application.

Suppose this servo must reduce the error resulting from a step input to a few per cent. of its initial value within one second. Then the period of the servo should be not greater than about $\frac{1}{3}$ second and ω_r must be at least 19. Therefore, to meet these requirements, the ratio $\frac{f}{J}$ should be about twenty, and the ratio $\frac{k}{f}$ should be about twenty-eight.

V. COMPARISON OF THE SERVOMECHANISM TO THE FEEDBACK AMPLIFIER.

Those familiar with negative feed amplifiers will have recognized the similarity between servomechanisms and feedback amplifiers by this point. Analytically these two devices are identical as is brought out by Fig. 8. Consequently many of the conclusions obtained for feedback amplifiers can be applied to servomechanisms. Nyquist [4] has established a most useful stability criterion for feedback amplifiers that is directly applicable to servomechanisms. In fact what has been termed the transfer locus is no more than the Nyquist diagram of the servomechanism.

The Nyquist stability criterion written for servomechanisms may be expressed as follows: Plot the locus of the transfer function of the system being studied and draw its conjugate (its mirror image referred to the

[4] Nyquist, H., "Regeneration Theory," *Bell System Technical Journal*, pp. 126–47, January, 1932.

real axis). If the combined curve encloses the point $(-1 + j0)$ the system is unstable. Otherwise the system is stable.

An extension of the criterion is required because of the fact that the transfer loci of most servomechanisms are of the open type—that is the locus and its conjugate do not join at zero frequency. Thus the locus drawn in Fig. 6 approaches $-90°$ at zero frequency and its conjugate approaches $+90°$. This difficulty can be remedied by a mathe-

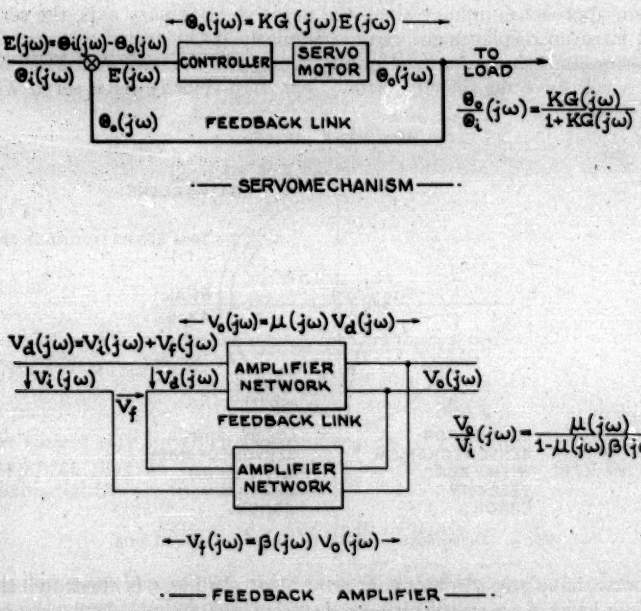


FIG. 8. Comparison of the servomechanism to the feedback amplifier.

matically sound artifice. If a curve of infinite radius is drawn to connect at zero frequency the locus and its conjugate in such a fashion that over the curve no discontinuity in phase occurs, the open type of diagram is transformed into a closed type of diagram and the previously stated criterion is directly applicable. Fig. 9 illustrates this procedure.

VI. FORM OF LOCUS FOR VARIOUS TYPES OF STEADY-STATE PERFORMANCE.

The performance of a servomechanism under steady-state conditions is always of great importance. If the servomechanism is primarily a positional device, it is desirable that the servomechanism take up various

positions without requiring an error to maintain it in that position. Similarly it is frequently necessary for the servomechanism to follow an input of constant velocity (that is, one in which $\theta_i(t) = kt$), without requiring a system error to maintain that velocity. Servomechanisms that satisfy the first condition frequently are termed zero-displacement-error servomechanisms and servos that meet the second condition are called zero-velocity-error servomechanisms.

It can be readily shown that if the transfer locus of a servomechanism approaches infinity along the negative imaginary axis, the servo will have no displacement error. Similarly if the transfer locus of a servomechanism approaches infinity along the negative real axis that servo will have no velocity error. The zero-velocity-error servo will,

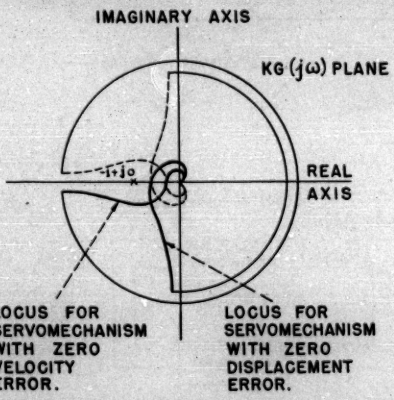


FIG. 9. Determination of the stability of loci of open form.

of course, have zero-displacement error also. In Fig. 9 is illustrated the transfer locus of a zero-velocity-error system and of a zero-displacement-error system. This concept can be extended to servo systems that will follow a constant input acceleration without steady-state error, and so forth.

The proof of the above relation for a zero-displacement-error servo is readily given. The Laplacian transform relationship between the servo output and input is obtained by replacing "$j\omega$" by "s" in Equation (13). Thus

$$\Theta_0(s) = \frac{KG(s)}{1 + KG(s)}\,\Theta_i(s). \tag{24}$$

In order to determine the requirements for a zero-positional-error-servo, the procedure is to let $\theta_i(t)$ be a step function of unit magnitude

and determine the condition necessary in order that the final (steady-state) magnitude of $\theta_0(t)$ be also of unit magnitude. If $\theta_i(t)$ is a step function of unit magnitude, its transform $\Theta_i(s)$ is equal to $\frac{1}{s}$. The steady-state value of $\theta_0(t)$ is readily found by making use of the final-value theorem [5] of Laplacian transform theory. This can be expressed in equation form as

$$\lim_{t\to\infty} \theta_0(t) = \lim_{s\to 0} s\Theta_0(s). \tag{25}$$

Therefore in order for a servo to have zero steady-state positional error,

$$\lim_{s\to 0} \frac{sKG(s)}{1 + KG(s)} \frac{1}{s} = 1, \tag{26}$$

or

$$\lim_{s\to 0} KG(s) = \lim_{s\to 0} [1 + KG(s)]. \tag{27}$$

This requirement is met only if

$$\lim_{s\to 0} KG(s) = \infty. \tag{28}$$

Translating this requirement back into frequency terms,

$$\lim_{j\omega\to 0} KG(j\omega) = \infty. \tag{29}$$

The easiest way in which this requirement is met is to so design the equipment that its transfer function at low frequencies may be approximated by

$$KG(j\omega) = \frac{k}{j\omega} \text{ for small values of } \omega. \tag{30}$$

The locus in the complex plane of such a system approaches infinity along the negative imaginary axis.

The derivation of the criterion for zero-velocity-error systems follows a similar line. In this case, the mathematical requirement for a system to have no steady-state error when the input is a constant velocity is

$$\lim_{j\omega\to 0} j\omega\, KG(j\omega) = \infty. \tag{31}$$

The simplest way in which this condition can be met is by requiring that

$$KG(j\omega) = \frac{k}{(j\omega)^2} \text{ for small values of } \omega. \tag{32}$$

[5] Gardner and Barnes, ref. cit.

The locus in the complex plane of such a system approaches infinity along the negative real axis.

VII. DYNAMICAL CHARACTERISTICS OF SERVOMECHANISMS.

The simplest transfer function that will fulfill the requirements of a zero-displacement-error servo is

$$K_b G_b(j\omega) = \frac{K_b}{j\omega}. \tag{33}$$

This function cannot be realized by a physical servomechanism since it requires such a combination of servo motor and servo controller that the output velocity of the motor is instantaneously proportional to the servo error.

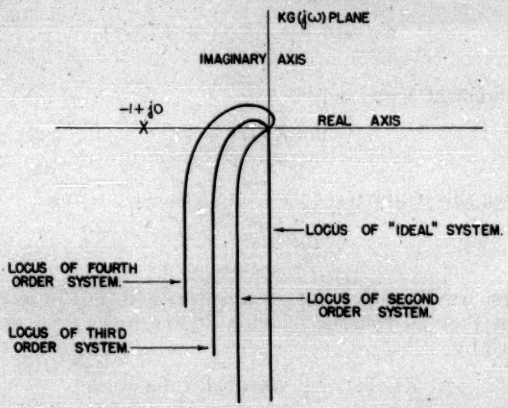


FIG. 10. Effect of the order of the servomechanism on its transfer function locus.

The transfer function of this ideal zero-displacement-error system is illustrated by Fig. 10. Its salient property is that since the locus lies along the negative real axis the scale of the plot can be increased without limit (meaning that the sensitivity, K_b, can be increased without limit) and the maximum value of $\left|\frac{\theta_0}{\theta_i}(j\omega)\right|$ will not change, but the frequency band width of the system will increase in proportion to the sensitivity. Thus the response of this hypothetical system can be made as fast as desired with no loss in stability simply by increasing the sensitivity. While such an ideal system cannot exist physically, its study is helpful as an indication of the direction along which design efforts should proceed.

Since a physical servomechanism comprises a number of electrical or mechanical systems coupled together, its transfer function is never as simple as that given above. Instead it contains a number of "lag" terms which cause the function to depart from the ideal function. For example, the transfer function of many physical servo systems can be represented by the following function:

$$KG(j\omega) = \frac{1}{j\omega(1 + j\omega a_1)(1 + j\omega a_2)(1 + j\omega a_3)\cdots}, \qquad (34)$$

in which the values of the parameters a_1, a_2, a_3, may be real or complex. Each of the quantities $\frac{1}{1 + j\omega a_1}$, $\frac{1}{1 + j\omega a_2}$, etc., introduces a phase lag into the system and each shifts the locus of the system farther from the ideal locus, $\frac{1}{j\omega}$, toward the point $-1 + j0$. The number of the parameters, a_1, a_2, a_3, etc., is proportional to the complexity of the system, and the number and size of the parameters determine the dynamical response of the system. The power of the frequency that approximates the transfer function at very high frequencies is known as the "order" of the system. Thus the transfer function given by Equation (18) is a second-order function, and the same function with two additional lag terms is a fourth-order function. Possible loci for systems of various order are illustrated in Fig. 10.

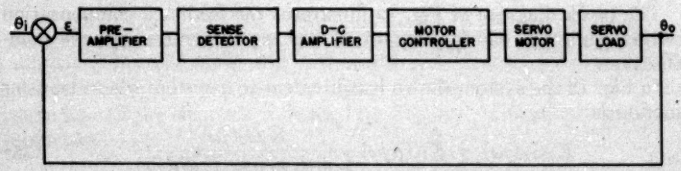


FIG. 11. Block diagram of a multi-lag system.

The system represented by the block diagram of Fig. 11 may have a transfer function of the type given by Equation (34). If each of the units, preamplifier, detector, motor controller, and servo motor has a sufficiently high effective input impedance that the characteristics of one unit do not affect the characteristics of the other, the overall transfer function of the system is the product of the transfer functions of the various components. In cases where this is not true the transfer function of the complete system is not the product of the separate transfer functions. In these cases the overall transfer function is a more involved function of the components. However, it can be factored into a form similar to Equation (34), but the individual terms may no longer be identified with only a single system component.

The performance of a specific system can be determined by plotting the function $G(j\omega)$, determining the maximum permissible sensitivity, K, for a reasonable maximum value of $\left|\frac{\theta_0}{\theta_i}(j\omega)\right|$, and the corresponding band width. From the value of band width and the permissible sensitivity a decision can be reached as to whether the response will be satisfactory for the application for which the system is intended.

VIII. DESIGN OF LAG-COMPENSATING NETWORKS.

If a study of the transfer function of a servomechanism shows that the dynamic response of the system is unsatisfactory for the application for which it is intended, an effort is made to so alter the design of the system that the parameters $a_1, a_2, \cdots$ are reduced to values that will provide a satisfactory transient response. In some cases it even may be practical to remove certain of the lag factors altogether. If it is found impractical to alter the system design to enable it to meet the transient requirements of the application, devices must be inserted into the system to compensate for the remaining lags to the extent demanded by the application.

While other means have been used of improving the transient response of a servomechanism, the two common methods are (1) to feed a signal into the servo controller proportional to some frequency function of the output and (2) to cascade devices with the controller that will compensate for lags in the servo motor or controller.

The block diagram of Fig. 12 illustrates the feedback compensation method while the block diagram of Fig. 13 illustrates cascade compensation.

In Fig. 12 the system shown is equivalent to a system whose transfer function is

$$K_T G_T(j\omega) = K_1 G_1(j\omega) \frac{K_2 G_2(j\omega)}{1 + K_2 G_2(j\omega) K_3 G_3(j\omega)}. \qquad (35)$$

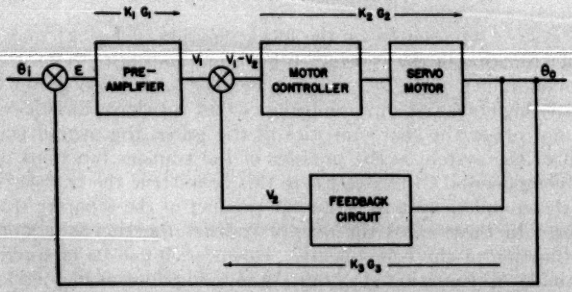


FIG. 12. Block diagram of a servomechanism with feedback compensation.

Since the transfer function of the system before the feedback correction was inserted was $K_1G_1(j\omega)K_2G_2(j\omega)$ it is evident that the transfer function has been altered by the factor $\dfrac{1}{1 + K_2G_2(j\omega)K_3G_3(j\omega)}$. The function $K_3G_3(j\omega)$ is chosen to make this factor properly compensate for lags in the original system.

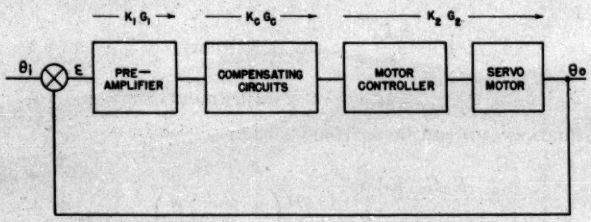


FIG. 13. Block diagram of a servomechanism with cascade compensation.

A simple example will illustrate the usefulness of the feedback method of lag compensation. The transfer function of the system illustrated by Fig. 2 has been given by Equation (18). This can be written as

$$KG(j\omega) = \frac{k_2}{f}\frac{1}{(j\omega)\left(1 + \dfrac{J}{f}j\omega\right)}. \tag{18a}$$

This corresponds to the function $K_2G_2(j\omega)$ of Fig. 12. If a preamplifier is added to the system of Fig. 2 and also a feedback circuit to add a signal proportional to the *rate of change* of the output with time, the system has all the elements indicated by Fig. 12. The equations of the system are

$$K_1G_1(j\omega) = k_1, \tag{36}$$

$$K_2G_2(j\omega) = \frac{k_2}{f}\frac{1}{j\omega\left(1 + \dfrac{J}{f}j\omega\right)}, \tag{37}$$

$$K_3G_3(j\omega) = j\omega k_3. \tag{38}$$

Equation (38) may be physically realized by coupling a generator to the output and using its voltage as the feedback signal.

The overall transfer function of this system, found by making use of Equation (35), is

$$K_TG_T(j\omega) = \frac{k_1k_2}{f\left(1 + \dfrac{k_2k_3}{f}\right)}\frac{1}{j\omega\left(1 + \dfrac{1}{1 + \dfrac{k_2k_3}{f}}\dfrac{J}{f}j\omega\right)}. \tag{39}$$

A comparison of the above equation with Equation 18a will show that the two transfer functions are of similar form but that the effect of the lag factor has been diminished by the factor $1 + \frac{k_2 k_3}{f}$. If the factor $1 + \frac{k_2 k_3}{f}$ is represented by r, and the gain of the preamplifier is adjusted to r^2,

$$1 + \frac{k_2 k_3}{f} = r \text{ (by definition)},$$

$$k_1 = r^2 \text{ (by adjustment)},$$

then Equation (39) can be written

$$K_T G_T(j\omega) = \frac{k_2}{f} \frac{1}{j\frac{\omega}{r}\left(1 + \frac{J}{f} j\frac{\omega}{r}\right)}. \tag{40}$$

Equation (40) is of exactly the form of Equation (18a) except for the fact that the frequency, ω, has been divided by r. This means that the frequency band width has been increased by r. Thus if the original system responded to a given input with a transient lasting one second, the feedback compensated system would respond to the same input in exactly the same fashion except the transient would last only $\frac{1}{r}$ second.

This example indicates the type of analysis that is employed and the type of results that may be expected when feedback compensation is employed. The type of function employed for the feedback path depends upon the system being corrected, and for more complex systems the analysis is not as simple as that given above.

If cascade compensation is employed rather than feedback compensation, the block diagram of the system is illustrated by Fig. 13. Here the transfer function of the overall system is the product of the transfer functions of the components. Thus

$$K_T G_T(j\omega) = K_1 G_1(j\omega) K_c G_c(j\omega) K_2 G_2(j\omega).$$

The manner in which cascade compensation can be employed may be illustrated by a similar example. Suppose cascade compensation is to be employed with the system illustrated by Fig. 2. Types of compensating circuits that can be employed are illustrated by Fig. 14.

The transfer functions of the various system components are

$$K_1 G_1(j\omega) = k_1, \tag{41}$$

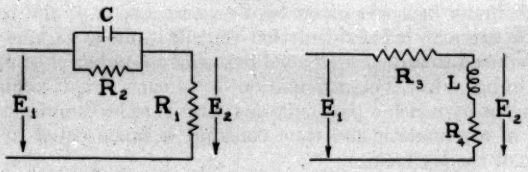
$$K_2 G_2(j\omega) = \frac{k_2}{f} \frac{1}{j\omega\left(1 + \frac{J}{f} j\omega\right)}, \tag{42}$$

$$K_c G_c(j\omega) = \frac{1}{\alpha}\,\frac{1 + j\omega\alpha\tau}{1 + j\omega\tau}, \tag{43}$$

in which

$$\alpha = \frac{R_1 + R_2}{R_1}, \tag{44}$$

$$\tau = \frac{R_1 R_2}{R_1 + R_2}\,C. \tag{45}$$



$$\alpha = \frac{R_1 + R_2}{R_1} \qquad \alpha = \frac{R_3 + R_4}{R_4}$$

$$\tau = \frac{R_1 R_2}{R_1 + R_2}C \qquad \tau = (R_3 + R_4)L$$

$$\frac{E_2}{E_1} = \frac{1}{\alpha}\,\frac{1 + j\omega\alpha\tau}{1 + j\omega\tau}$$

FIG. 14. Basic phase-lead networks.

If the time constant of the compensating circuit is so adjusted that

$$\alpha\tau = \frac{J}{f}, \tag{46}$$

the overall transfer function may be written

$$K_T G_T(j\omega) = \frac{k_1 k_2}{\alpha f}\,\frac{1}{j\omega\left(1 + \frac{J}{f\alpha}\,j\omega\right)}. \tag{47}$$

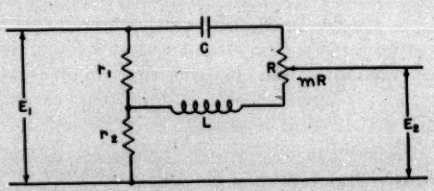
A comparison of the above equation with the original Equation (18a) for the system before the compensating circuit was added reveals that the effect of the lag term has been reduced by the factor α. If the gain of the preamplifier is adjusted to equal α^2, Equation (47) may be written

$$K_T G_T(j\omega) = \frac{k_2}{f}\,\frac{1}{j\frac{\omega}{\alpha}\left(1 + \frac{J}{f}\,j\frac{\omega}{\alpha}\right)}. \tag{48}$$

A comparison of expressions (18a) and (48) shows that the two have similar form but that the band width has been increased by a factor α. Therefore, if the uncompensated system required one second to diminish a transient arising from a particular input to a certain percentage of its original value the compensated system would require only $\frac{1}{\alpha}$ second to reduce the transient arising from the same input to the same percentage of its original value.

A general type of transfer-function for the zero-positional-error system with many lags was given by Equation (34). If the parameters $a_1, a_2, \cdots$ are real, it is evident that the effect of the various lag terms can be reduced to any desired level by using a number of compensating circuits in cascade. The attenuation, α, of each circuit is made equal to the factor by which a particular lag term is to be diminished and the product of attenuation and time constant is made equal to the time constant of the lag term.

In case certain of the terms a_1, a_2, etc., are complex, the procedure must be somewhat modified. When complex terms exist, they must occur in conjugate pairs and the product of the pair can be written $\frac{1}{1 + (a_1 + a_2)j\omega - a_1 a_2 \omega^2}$ in which the sum, $(a_1 + a_2)$, and the product $a_1 a_2$ are real. It is now necessary to find a compensating circuit whose transfer function is the complement of this second-order lag term. The circuit of Fig. 15 is such a circuit. The relations given in the figure hold



$$\frac{E_1}{E_2} = \frac{(j\omega)^2 + 2\zeta_c \omega_c j\omega + \omega_c^2}{(j\omega)^2 + 2\zeta_c n\omega_c j\omega + \alpha_c^2 \omega_c}$$

$$\alpha_c^2 = \frac{r_1 + r_2}{r_2} \qquad n = \frac{\alpha_c^2}{1 + m(\alpha_c^2 - 1)}$$

$$\omega_c = \frac{1}{\alpha_c^2 LC} \qquad \zeta_c = \frac{1}{2} \frac{R}{nL\omega_c}$$

FIG. 15. Complex phase-lead network.

provided r_1 and r_2 are small compared with R. If r_1 and r_2 are not negligible the relations are somewhat complicated but the circuit still has a transfer function of the form given in the figure.

IX. DESIGN OF CIRCUITS FOR MINIMIZING STEADY-STATE ERRORS.

It is frequently necessary to design circuits that will change a zero-positional-error servo into a zero-velocity-error system. This may be accomplished by several methods, but the method described here employs a circuit that is cascaded with the controller. Thus if the block diagram of Fig. 13 applies to a zero-positional-error system, the problem is to design a compensating circuit with a transfer-function K_cG_c that will transform the system to a zero-velocity-error servo.

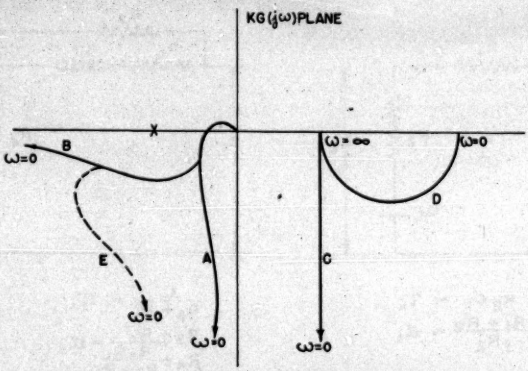


FIG. 16. Transfer loci of integral control system.

The type of circuit required may be determined by a study of system transfer loci. Curve A of Fig. 16 is the transfer locus of a typical zero-positional-error system and curve B is the locus of a corresponding zero-velocity-error system. The problem is to obtain a physical realizable function that when multiplied by the transfer function of curve A will result in a locus of the type illustrated by curve B. The function characteristics of the system will be affected unfavorably.

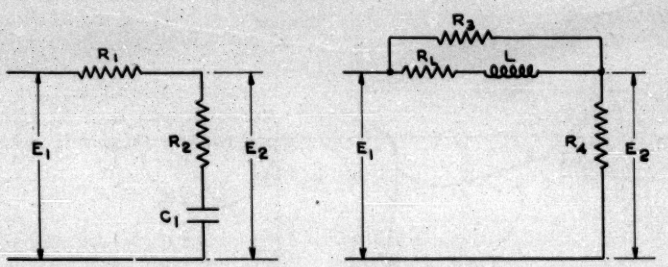
The type of circuit required may be determined by a study of system transfer loci.

The locus of the simplest function that will accomplish this is plotted as curve C in Fig. 16. The function itself is $k\left(1+\frac{1}{j\omega\tau_i}\right)$. It is easily shown that this function is the transfer function of a device whose output is proportional to the sum of the input and the time integral of

the input. It is thus commonly known as an "ideal-integral-response device."

While an ideal integral response can be obtained only by a regenerative element, it can be approximated as closely as desired by a passive circuit in cascade with an amplifier. Figure 17 illustrates two possible types of circuits.

The transfer locus of both of these circuits is a semi-circle as illustrated by curve D of Fig. 16. The semi-circle closely approximates the straight-line locus, curve C, at higher frequencies, at which the two loci approach tangency. The larger the semi-circle the better is the approximation between the two curves. The diameter of the semi-circle is proportional to the attenuation factor α_i.



$$R_2 C_1 = \tau_i \qquad \frac{R_1 + R_2}{R_2} = \alpha_i$$

$$\frac{L}{R_3 + R_L} = \tau_i \qquad \frac{R_3 + R_4}{R_4 + \frac{R_3 R_L}{R_3 + R_L}} = \alpha_i$$

$$\frac{E_2}{E_1} = \frac{1 + \tau_i j\omega}{1 + \alpha_i \tau_i j\omega}$$

FIG. 17. Integral control networks.

The transfer locus of a servomechanism employing such a compensating circuit will resemble locus B of Fig. 16 except at very low frequencies where it departs from curve B and follows a curve such as curve E. Since the locus approaches the negative imaginary axis as the frequency approaches zero, the system is not a true zero-velocity-error system. However, the velocity error of the original system is diminished by the attenuation factor, α_i, of the compensating systems. For many applications an α_i of about ten reduces the velocity error to an acceptably small value.

The time constant, τ_i, is made as small as possible without diminishing the damping characteristics of the system. If the time constant is too small, the locus approaches too close to the unity point. If the time constant is too long, the system requires too long to correct an error. A rule that is applicable to the majority of cases is to so choose the time constant τ_i, that the phase shift introduced by the integral control circuit is about five or ten degrees at the resonant frequency of the system.

ACKNOWLEDGMENT.

The author gratefully acknowledges the assistance of his colleagues at the Servomechanisms Laboratory, M.I.T., in which much of this work was done. Thanks are due particularly to Dr. G. S. Brown, director of the laboratory, for providing opportunities to apply the theory to practical servo design problems.

Public Lands Show Increase in Production of Minerals.—Director W. E. Wrather of the Geological Survey informed Secretary of the Interior J. A. Krug that the number of public-land mineral leases in force, the area covered by such leases, and the production of petroleum, natural gas, potassium, sodium, phosphate rock, and silica sand on leased public lands increased substantially during the fiscal year ending June 30, 1946. The recent strike, which paralyzed the coal industry and severely hampered production throughout the United States, resulted in only a small decrease in coal production on public lands.

Under supervision of the Conservation Branch of the Geological Survey, the recovery of minerals, including oil and gas, from 9,121 leases embracing 6,168,438 acres amounted to 59,567,525 barrels of petroleum; 96,070,275,000 cubic feet of natural gas; 120,819,733 gallons of gasoline and butane; 9,282,059 tons of coal; 3,919,739 tons of potassium salts; 276,381 tons of sodium salts; 23,355 tons of silica sand; and 91,650 long tons of phosphate rock.

This compares with the fiscal year 1945, during which the production from 7,335 leases embracing 4,652,908 acres amounted to 55,907,088 barrels of petroleum; 95,920,557,000 cubic feet of natural gas; 127,922,745 gallons of gasoline and butane; 10,117,328 tons of coal; 3,775,860 tons of potassium salts; 243,614 tons of sodium salts; 73,360 long tons of phosphate rock; and 15,000 tons of silica sand.

Dr. Wrather pointed out that the increases in production of petroleum as well as other minerals sharply reflect the results of the Department's all-out effort to produce minerals for war and reveal, in part, the important rôle of the Conservation Branch in this program. Had the war continued, it is likely that the increases in production would have been materially greater.

R. H. O.

"Power Plus" for the Amador Central Railroad.—"Power Plus," are the words of the managing superintendent of Amador Central Railroad for their new ALCO-G.E. 44-ton diesel-electric locomotive. In operation since June, 1945, this diesel-electric, replacing a steamer, has been available 99 per cent. of the time, and resulted in savings of 75 per cent. in fuel costs. Maintenance costs have been unusually low. "Efficient, dependable, rugged, and economical," is another Amador official's statement on this locomotive.

Principally a forest products road, the Amador Central also hauls finished lumber, oil, copper and gold ore, fire brick, lime rock, and clay shipments. This short line of 11.8 miles, from Martel to Ione, Calif., has grades up to 4 per cent. and 23-degree curves with 252-degree curvature per mile of road. To accommodate an altitude drop of 1,200 feet, there are 11 trestles along the route, the largest 68 feet high and 375 feet long.

Operation performance of the diesel-electric locomotive is being closely watched by operators in the lumber and oil industries, and short line railroads. Amador's figures show that with the steam locomotive working on an eight-hour shift over a period of nine months, total operating costs less labor were $4,076.35. The 44-ton diesel-electric total operating costs less labor in a nine months period, but on a 20-hour shift were only $2,312.19. Moreover, the diesel-electric required refueling only once every 48 hours; the steam locomotive required refueling every trip.

R. H. O.

THE MAXIMUM INERTIA FORCE AND MAXIMUM SPEED OF THE CROSSHEAD IN RECIPROCATING ENGINES.

BY

W. S. KIMBALL, Ph.D.

Lt. Comdr., USNR, U. S. Naval Academy, Annapolis, Maryland.

The maximum speed and acceleration of the piston crosshead in reciprocating engines are here studied as functions of two variables, the angular position of the crank arm in a given engine cycle, and secondly, the ratio of the crank arm, r, to the connecting rod length, c, corresponding to a series of types of engines. The maximum acceleration or inertia force always occurs for practical engineering cases at the inside position of the stroke where the piston is withdrawn into the cylinder and is given by $-\omega^2 r[1 + (r/c)]$. Its form shows that r/c, the ratio of crank arm to connecting rod, is the percentage increase of this maximum acceleration over that for uniform circular motion having angular velocity ω, which is that of the crank arm.

For extreme cases outside of ordinary engineering practice where r/c ranges from .9 to unity and the connecting rod, c, scarcely exceeds the crank arm, r, in length, the maximum acceleration occurs where the crank arm is at right angles to the cylinder axis, $\theta = \pi/2$ in Fig. 1. Here the acceleration is $\omega^2 r^2/\sqrt{c^2 - r^2}$ and is seen to become infinite when $r = c$ and the ratio r/c is unity.

The maximum *velocity* occurs where this acceleration is zero and this is always *between* the position where the crank arm r is at right angles to the cylinder axis, with $\theta = \pi/2$, and the position where the crank arm r is at right angles to the *connecting rod* c, where the latter would be tangent to a flywheel. This result is due to the angularity of the connecting rod. In fact the angular position for maximum crosshead velocity is for practical cases within a degree or two of the position where r and c are at right angles. And the velocity at this latter tangent position falls short of the maximum by less than one per cent., and their graphs are almost indistinguishable over the practical engineering ranges of r/c, where this ratio is less than .5. See Fig. 5.

As a function of r/c, the crosshead motion here discussed is a modified simple harmonic motion—modified by the angularity of the connecting rod. Also it appears that the two limiting cases, between which all actual motions are included, are true simple harmonic motions. The first when $r/c = 0$, occurs for an infinite connecting rod, and the piston crosshead's motion is the same as for that of the projection of the crank end of uniform angular velocity on the axis line through the cylinder,

i.e., it is S.H.M. with velocity, $-\omega r \sin\theta$. The second limiting case at the other end of the range where $r/c = 1$, is also simple harmonic motion, but this time the velocity is double that for the previous limiting case, i.e., $V = -2\omega r \sin\theta$. Moreover this velocity occurs only during the inner half of the engine cycle. Where the crank arm is covering the half circle away from the cylinder, the crosshead stays at rest at the origin (the crank shaft axis) and the sudden drop in velocity from $-2\omega r$ to zero, entailed when the crank arm crosses the Y axis, accounts for the infinite acceleration there, for this purely hypothetical case.

The critical angular positions corresponding to maximum cross head velocities are determined by a cubic equation:

$$x^3 - x^2 - \rho x + \rho^2 = 0, \tag{13}$$

where $x = \csc^2\theta_c$, and θ_c is the critical angular position of the crank arm, and $\rho = r^2/c^2$ is the square of the ratio of crank arm to connecting rod.

Two aspects of the solutions of this equation are of interest. First, it is the reciprocal root equation corresponding to a previous equation (11) obtained more directly. By using the reciprocal root equation, the only root of any physical interest becomes the largest root, rather than the smallest one, and so is obtained by Graeffe's method of solution very much more easily than it would be if Graeffe's method were used to obtain a series of the smallest roots of (11).

Secondly (13), being a special type of cubic, is easily solved by setting $x = n\rho$, whereby ρ, and the critical quantities determined by (13) are readily expressed in terms of n. The maximum velocity of the crosshead so obtained is

$$V_m = -\frac{\omega r(n+1)}{\sqrt{n^2+n-1}} \tag{22}$$

and this is compared with the velocity where crank arm and connecting rod are at right-angles given by

$$V_t = -\omega r\sqrt{1+\rho} \tag{24}$$

with $\rho = \dfrac{n^2+n-1}{n^3}$ according to (13). These two velocities are found for practical engineering conditions $[(r/c) < .5]$ to be almost indistinguishable on the graph (Fig. 4) and to differ by less than one per cent.

The smallest critical angle θ_c is seen to be given by $n = 2$ (set the derivative of x with respect to ρ in (13) equal to zero). Then $V_m = -(3/\sqrt{5})\omega r$, and $\rho = r^2/c^2 = 5/8$, which gives a ratio of crank arm to connecting rod, too large for practical use. For this ρ and position θ_c, the angularity of the connecting rod (angle between c and axis of cylinder) is found to be 45°. See Eq. (19) and Fig. 3.

The present investigation differs from John L. Bogert's [1] in that here attention is focused on the *maximum* velocities and *maximum* accelerations over the entire range of possible values for r/c the ratio of crank arm to connecting rod, rather than giving entire velocity and acceleration curves for isolated cases. For the six isolated cases treated by Bogert, the acceleration maxima check the present values exactly as to position and magnitude, and the velocity maxima as to magnitude and as to position within two seconds of arc. The present treatment also compares the magnitude and position of velocity maxima, with magnitude and position of velocities with crank arm and connecting rod at right angles, showing the latter to differ from the former through practical ranges by less than one per cent. in magnitude and one degree in position.

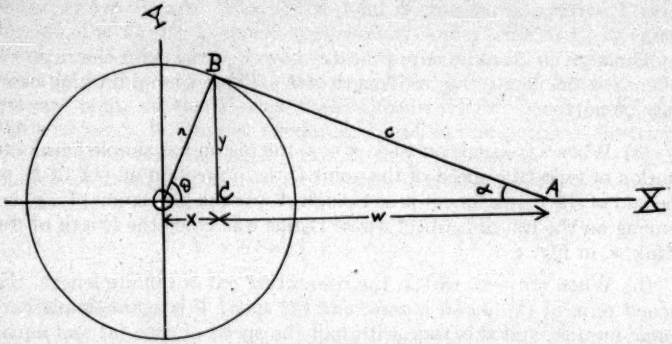


FIG. 1. The crosshead position A, as a function of the crank arm r, and connecting rod c, and the independent angle variable θ.

NO. 1. PISTON SPEED IN RECIPROCATING ENGINES AS A FUNCTION OF CRANK ANGLE AND THE RATIO OF CRANK ARM TO CONNECTING ROD.

We here make a study of the speed of the piston in ordinary reciprocating engines, having constant angular velocity ω for the crank shaft, or flywheel in case there is one.

Taking the center of the crankshaft as origin of coordinates, we have the distance to A on the piston p given as shown in Fig. 1 by:

$$OA = x + w = \sqrt{r^2 - y^2} + \sqrt{c^2 - y^2}, \tag{1}$$

where $x = OC = \sqrt{r^2 - y^2}$ and $w = CA = \sqrt{c^2 - y^2}$ are the projections

[1] Piston Displacement, Velocity and Acceleration for Reciprocating Engines," *International Marine Engineering*, **25**, 983, Dec. 1920.

on the x axis of the constant crank arm, $r = OB$ and constant connecting rod, $c = BA$.

The velocity of the piston at A is given by the time derivative of (1), introducing $y = r \sin \theta$, and $\omega = d\theta/dt =$ constant, where θ is the crank angle, and ω, the constant rotation of the crank shaft.

$$V = -y\frac{dy}{dt}\left(\frac{1}{\sqrt{r^2 - y^2}} + \frac{1}{\sqrt{c^2 - y^2}}\right) = -\omega r \sin\theta\cos\theta$$

$$\times\left(\frac{1}{\cos\theta} + \frac{r}{\sqrt{c^2 - r^2\sin^2\theta}}\right) = -\omega r \sin\theta$$

$$\times\left(1 + \frac{r}{c}\frac{\cos\theta}{\sqrt{1 - \frac{r^2}{c^2}\sin^2\theta}}\right) = \frac{dx}{dt} + \frac{dw}{dt}. \quad (2)$$

For a given crank r, note from (2) how V varies with the ratio r/c where c is the connecting rod length AB. Three special limiting cases may be noted:

(a) When r/c equals one, i.e., $r = c$, the piston has simple harmonic motion of twice the speed of the point C the projection on OX of B, at the end of the crank, i.e., it is as though A was the projection of a point moving on the rim of a wheel whose radius was twice the length of the crank, r, in Fig. 1.

(b) When $r/c = 0$, with c, the connecting rod of infinite length, the second term of (2), dw/dt is zero, and the speed V is again simple harmonic motion, and this time with half the speed of case (a) and equal to that of the projection point C of B on OX.

(c) When $r/c > 1/2$, the point A crosses the right-hand rim of the circle as B rounds the circuit at the left. This condition is outside of usual engineering practice, but may be provided by a laboratory model.

NO. 2. ACCELERATION OF THE PISTON WHERE THE CONNECTING ROD, c, IS AT RIGHT ANGLES TO THE CRANK OR CROSSES THE COORDINATE AXES.

Although dx/dt and dw/dt of (2) are, as implicit functions, always added, they experience different sign reversals as B crosses the coordinate axes. Thus $dx/dt = -\omega r \sin\theta$ is positive in the third and fourth quadrants and negative above OX in the first two quadrants where $y = r \sin\theta$ is plus. On the other hand dw/dt, which includes the factor $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$, experiences sign reversal four times in making the circuit, being positive in the second and fourth quadrants and negative in the first and third. This situation may be shown in tabular form as follows:

TABLE I.

Quadrant	I	II	III	IV
$\frac{dx}{dt} = x'$	−	−	+	+
$\frac{dw}{dt} = w'$	−	+	−	+

Table I shows that on the right of the Y axis the two velocity components dx/dt and dw/dt support each other (same sign) and on the left (II and III quadrants) the two components oppose each other. Note the implications of this for Case (a) No. 1: For this case, which is one of simple harmonic motion, we have maximum velocity given by (2) as $\mp 2\omega r$, at $\theta = \pm \pi/2$. But as the point B crosses the positive Y axis, the sign of w' (Table I) experiences reversal, and w' cancels x', its equal in magnitude for this case, so that velocity maximum $V = -2\omega r$ drops instantly to zero and remains zero until $\theta = 3(\pi/2)$ and B reaches the opposite point on the Y axes, then with an instantaneous jump, the piston velocity V resumes maximum speed in the positive direction. We may show that the acceleration at these points were $\theta = \pm \pi/2$ is infinite for case (a). Thus take the time derivative of (2) which may be written:

$$V = -\omega y\left(1 + \frac{\sqrt{r^2 - y^2}}{\sqrt{c^2 - y^2}}\right). \tag{3}$$

The time derivative of (2) or (3) is

$$V' = \frac{dV}{dt} = -\omega\frac{dy}{dt}\left[1 + \frac{y^4 - 2c^2y^2 + c^2r^2}{\sqrt{(r^2 - y^2)(c^2 - y^2)^3}}\right]$$

$$= -\omega^2 r\cos\theta\left[1 + \frac{c^2r^2\cos^2\theta - r^2\sin^2\theta(c^2 - r^2\sin^2\theta)}{r\cos\theta(c^2 - r^2\sin^2\theta)^{3/2}}\right]$$

$$= -\omega^2\left[\frac{r\cos\theta(c^2 - r^2\sin^2\theta)^{3/2} + c^2r^2\cos^2\theta - r^2\sin^2\theta(c^2 - r^2\sin^2\theta)}{(c^2 - r^2\sin^2\theta)^{3/2}}\right]. \tag{4}$$

This gives the acceleration of the piston at A as a function of $y = r\sin\theta$, the ordinate of B, or of θ the angular position of B, where the connecting rod is attached to the crank. Equations (2) and (4) are well known [2] expressions for piston velocity and acceleration and we now proceed to investigate their maximum values.

When the point B crosses the Y axis $\cos\theta = 0$ and $\sin\theta = \pm 1$,

[2] See "Kinematics of Machines" by G. L. Guillet (1930), Eqs. 5-9 and 5-11, p. 62.

hence from (4)

$$V' = \frac{\omega^2 r^2}{\sqrt{c^2 - r^2}}. \tag{5}$$

This is always positive, and for Case (a), which has $r = c$, it becomes infinite as noted above.

When B crosses the X axis, $\sin\theta = 0$ and $\cos\theta = \pm 1$. Hence we have from (4):

$$V' = -\omega^2 r\left(1 + \frac{r}{c}\right), \quad \theta = 2n\pi, \quad \cos\theta = 1, \tag{6}$$

$$V' = +\omega^2 r\left(1 - \frac{r}{c}\right), \quad \theta = (2n+1)\pi, \quad \cos\theta = -1. \tag{7}$$

Equation (6) shows that with B on the X axis to the right of the Y axis, the *acceleration* is negative and (7) that with B on the X axis to the left of the Y axis it is positive and smaller than with (6). Also (7) shows that for Case (a) above, V' is zero when B travels left of the Y axis (quadrants II and III) around at constant speed along the rim of the circle. During this left-hand half circle route of B, the point A at the cross head of the piston whose velocity and acceleration we refer to as V and V' herein, remains fixed at the origin and connecting rod c lies right over and along r (because $c = r$ for Case (a)). For practical engineering, where c is from 3 to 5 times as great as r, the *maximum acceleration* of the piston A is at the inside withdrawn position of the stroke where $\theta = 2\pi n$ as given by (6).

When the connecting rod c is tangent to rim of the wheel or circle shown in Fig. 1, the angle between r and c is 90°, and angle α which measures the angularity of the connecting rod, is the complement of θ. Thus from Fig. 1

$$\left.\begin{aligned} \pm\sin\theta = \cos\alpha &= \frac{c}{\sqrt{c^2 + r^2}} \\ \pm\cos\theta = \sin\alpha &= \frac{r}{\sqrt{c^2 + r^2}} \\ c^2 - r^2\sin^2\theta = c^2\sin^2\theta &= \frac{c^4}{c^2 + r^2} \end{aligned}\right\} 0 < \alpha < \frac{\pi}{2}. \tag{8}$$

When (8) are introduced into (4) we have:

$$V' = -\omega^2\frac{r^4}{c^4}\sqrt{c^2 + r^2}; \quad \pm\sin\theta = \cos\alpha = \frac{c}{\sqrt{c^2 + r^2}} \; (r \perp \text{to } c) \tag{9}$$

for both positive and negative values of $\sin\theta$. Since (5) and (9) have opposite signs, we see that maximum velocity, where V' is zero, occurs

between the right angle position of the connecting rod (angle at B is 90°, where it is tangent to the circle of Fig. 1) given by (9) and where its end crosses the Y axis with $\theta = (2n + 1)\frac{\pi}{2}$.

NO. 3. THE INERTIA FORCES DUE TO PISTON ACCELERATION.

From the engineering viewpoint the force due to rate of change of momentum is of importance, because with a heavy piston, the inertia involved where high rates of change of speed occur, may figure as a major factor in the wear and tear of the engine.

We have already noted that for Case (a), section 1, with connecting rod c, equal to crank arm, $r(r = c)$, this acceleration and momentum change are infinite at the point where B, the end of the connecting rod, crosses the Y axis. This is, of course, an engineering impossibility, but on the other hand, if c, the connecting rod length, approaches r, the crank length, the forces of inertia will be enormous and a corresponding bad engineering situation will be represented. This also, however, will probably never be a serious consideration because with ordinary engineering practice, where the ratio of crank arm to connecting rod ranges from 1/3 to 1/5, the inertia effect due to acceleration is greatest at the inside withdrawn position of the stroke where $\theta = 2\pi n$ and $\cos\theta = 1$ as pointed out in connection with (6). Even when this ratio is as large as 4/5, i.e., $r = .8c$ the acceleration at $\theta = 90°$ given by (5) is only $4/3\omega^2 r$, whereas for this same $r = .8c$ at $\theta = 0$, (6) becomes $-9/5\omega^2 r$ which is only 10 per cent. less than its limiting value $-2\omega^2 r$, that holds when r/c = one, and (5) is infinity. See Fig. 2 and Table II below.

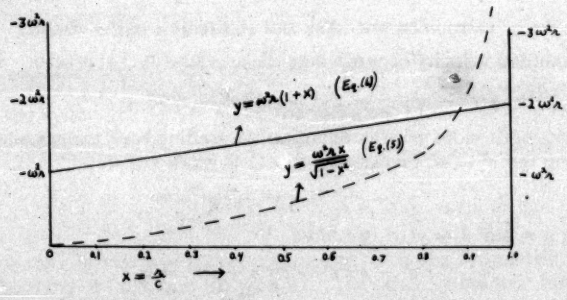


FIG. 2. Curves of relative maximum crosshead acceleration as a function of r/c, the ratio of crank arm to connecting rod, showing the absolute maximum through the range of engineering practice at $\theta = 0$ for Eq. (6), but with the absolute maximum at $\theta = \pm\pi/2$ by Eq. (5) for extreme values of r/c beyond the critical 0.883 according to the dashed curve.

TABLE II.

$x = r/c$	0	1/5	1/4	1/3	.5 = 1/2	3/5 = .6	4/5 = .8	9/10 = .9	24/25 = .96
Eq. (5) $\frac{\omega^2 r^2}{\sqrt{c^2 - r^2}}$	0	$\frac{\omega^2 r}{\sqrt{24}}$	$\frac{\omega^2 r}{\sqrt{15}}$	$\frac{\omega^2 r}{2\sqrt{2}}$	$\frac{\omega^2 r}{\sqrt{3}}$	$3/4\omega^2 r$	$4/3\omega^2 r$	$\frac{9\omega^2 r}{\sqrt{19}}$	$24/7\omega^2 r$
Eq. (6) $\omega^2 r\left(1+\frac{r}{c}\right)$	$\omega^2 r$	$6/5\omega^2 r$ $1.2\omega^2 r$	$5/4\omega^2 r$ $1.25\omega^2 r$	$4/3\omega^2 r$ $1.33\omega^2 r$	$3/2\omega^2 r$ $1.5\omega^2 r$	$8/5\omega^2 r$ $1.6\omega^2 r$	$9/5\omega^2 r$ $1.8\omega^2 r$	$19/10\omega^2 r$ $1.9\omega^2 r$	$49/25\omega^2 r$ $1.96\omega^2 r$

The above figures refer to relative maxima of acceleration (and hence inertia force) during the engine cycle. We may plot these acceleration maxima given by (5) and (6) against the ratio of crank arm to connecting rod, r/c, as this ratio varies from 0 to 1 between the two limiting cases (b) and (a) of section one. Figure 2 shows the graphs of these two relative maxima, corresponding to their tabulated values given in Table II. The linear graph is Equation (6) (absolute value) and the dash line curve is the graph of (5), with the independent variable taken as $x = r/c$, the ratio of crank arm to connecting rod. Note that for practical engineering where x is small the absolute maxima are given by (6) at the inside piston position where $\theta = 2n\pi$ and $\cos\theta = +1$. But for x larger than the critical value .883 (given by setting (5) equal to (6) (absolute value) and solving for $x = r/c$) the absolute maxima occur at $\theta = (2n+1)(\pi/2)$ with $\cos\theta = 0$. Thus, as $x = r/c$ ranges from 9/10 to 1, (5) gives the absolute maxima, and these increase rapidly beyond $2\omega^2 r$ up to infinity.

The lower row of values in Table II shows that for x smaller than the critical .883 (practical engineering) the percentage increase in maximum crosshead inertia force per unit mass given by (6) over that at the other end B of the connecting rod (always given by $-\omega^2 r$) is exactly the ratio, $x = r/c$, of crank arm to connecting rod.

NO. 4. POSITION OF THE CRANK ARM AT MAXIMUM PISTON VELOCITY.

Maximum velocity occurs where the acceleration (4) is zero. That is

$$\sqrt{(r^2 - y^2)(c^2 - y^2)^3} = -(y^4 - 2c^2y^2 + c^2r^2).$$

Squaring both sides of this equation, cancelling and transposing, and factoring out $c^2 - r^2$, gives:

$$y^6 - c^2y^4 - c^4y^2 + c^4r^2 = 0. \tag{10}$$

Setting $z = \sin^2\theta = y^2/r^2$, we have

$$r^6z^3 - c^2r^4z^2 - c^4r^2z + c^4r^2 = 0.$$

Divide by c^4r^2 and obtain

$$\rho^2z^3 - \rho z^2 - z + 1 = 0, \qquad \text{where} \qquad \rho = \frac{r^2}{c^2}. \tag{11}$$

This equation has three real roots, as may be easily shown graphically, taking ρ as .1 or less for actual engineering problems. One of these roots is negative, which has no meaning for us since $z = \sin^2\theta$. Of the two positive roots, one is greater and one less than unity. In view of our definition of z, the latter is the only one of any significance for us, and it is the root of smallest magnitude. Hence it is the last and most laborious one of the three to find by Graeffe's method, which is characterized by successive evaluation of roots from the largest in magnitude down to the least. We wish to obtain a series of solutions of (11) for various values of ρ. Hence this difficulty is a serious one as applied to (11). But it is not inherent in Graeffe's method since the equation whose roots are the reciprocals of the roots of (11) may be easily set up, and its largest root will be the reciprocal of the smallest of (11), and, being the largest one, is the first one obtained by Graeffe's method. It will be the reciprocal of z or $\csc^2\theta$ where θ is the critical angle that determines where the velocity (2) is maximum.

The reciprocal root equation corresponding to (11) is given by introducing a variable which is the reciprocal of z of (11):

$$x = 1/z = 1/\sin^2\theta = \csc^2\theta. \tag{12}$$

Substituting (12) in (11) gives:

$$f(x) = x^3 - x^2 - \rho x + \rho^2 = 0. \tag{13}$$

That (13) has roots that are the reciprocals of the roots of (11) is proved by the method of obtaining (13). The relation between the symmetric functions of the roots of (11) and (13) becomes more clear if we illustrate by a more general equation. Thus the quartic:

$$z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0 \tag{14}$$

becomes under the transformation (12):

$$x^4 + \frac{a_3}{a_4}x^3 + \frac{a_2}{a_4}x^2 + \frac{a_1}{a_4}x + \frac{1}{a_4} = 0 \tag{15}$$

and the symmetric functions of the roots of (15) are readily seen to be expressible in terms of roots that are the reciprocals of the roots of (14). Thus the sum of the roots of (15) is given by:

$$-\frac{a_3}{a_4} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4} = x_1 + x_2 + x_3 + x_4,$$

which may be checked by expressing the a's as the symmetric functions of the roots of (14), and likewise with the other symmetric functions of the roots of (15).

We have pointed out in connection with (9) that the maximum velocity (2) occurs between the position of right angles between crank arm and connecting rod given by (9) and the position given by (5)

where B of Fig. 1, crosses the Y axis at $\theta = \pi/2$, because (5) and (9) give accelerations of opposite sign. This conclusion is checked by (13). Thus take $\theta = \pi/2$ corresponding to (5) with $x = \csc^2 \pi/2 = 1$. Then (13) becomes:

$$f(1) = -\rho + \rho^2 < 0 \qquad \text{since} \qquad \rho < 1. \tag{16}$$

And for the situation of (9) we have from (8) $\csc^2\theta = \dfrac{r^2 + c^2}{c^2} = \rho + 1$,

which when substituted for x in (13) gives

$$f(1 + \rho) = \rho^3 + 2\rho^2 > 0. \tag{17}$$

Thus $f(x)$ changes sign from plus to minus as the angle θ goes from the tangent to the circle position in Fig. 1 for the connecting rod, according to (17) where $x = 1 + \rho$ to the point where its end B crosses the Y axis according to (16) where $x = 1$. Accordingly the zeros of $f(x)$ given by (13) where the maximum of the piston speed (2) occurs are between these two positions.

Using Graeffe's method we may tabulate the coefficients of the cubic equations whose roots are the indicated powers of the roots of (13):

TABLE III.

The Four Coefficients of the Cubics Whose Roots are the Indicated Powers of the Roots of (13).

1st power	1	$a_1 = -1$	$a_2 = -\rho$	$a_3 = \rho^2$
	1	$1 + 2\rho$	$\rho^2 + 2\rho^3$	ρ^4
2nd power	1	$2\rho + 1$	$3\rho^2$	ρ^4
	1	$(2\rho + 1)^2 - 6\rho^2$	$9\rho^4 - 2\rho^4(2\rho + 1)$	ρ^8
4th power	1	$1 + 4\rho - 2\rho^3$	$7\rho^4 - 4\rho^5$	ρ^8
	1	$(1 + 4\rho - 2\rho^2)^2 - 2(7\rho^4 - 4\rho^5)$	$(7\rho^4 - 4\rho^5)^2 - 2\rho^8(1 + 4\rho - 2\rho^2)$	ρ^{16}
8th power	1	$1 + 8\rho + 12\rho^2 - 16\rho^3 - 10\rho^4 + 8\rho^5$	$\rho^8(47 - 64\rho + 20\rho^2)$	ρ^{16}

$$\rho = 1/10 \begin{cases} x_1^8 = 1 + 8\rho + 12\rho^2 - 16\rho^3 - 10\rho^4 + 8\rho^5 \\ x_1^8 = 1 + .8 + .12 - .016 - .001 + .00008 \\ x_1^8 = 1.90308 \end{cases}$$

$$\begin{aligned} &\log x_1 = 1/8 \log 1.90308 = 1/8(.27962) \\ &\log \csc\theta = 1/16(.27962) = .017476 \\ &\log \sin\theta = 9.982524-10 \\ &\sin\theta = .96056 \qquad \theta = 73°51'. \end{aligned}$$

When the roots are those of (13) raised to the 8th power, it is only the largest of them, x_1^8, that need be considered in the sum of the roots given by the 2nd column since the third column is then of the order $\rho^8 = 10^{-8}$. (See Doherty and Keller, "Mathematics of Modern Engineering," § 46, p. 107.)

We may tabulate the solutions of (13) by Graeffe's method for a few crank arm vs. connecting rod ratios covering the usual range of values used in engineering:

TABLE IV.

r/c	$\rho = \frac{r^2}{c^2}$	$x_1^8 = \csc^{16}\theta_m$	$\sin\theta_m$	θ_m
1/2	1/4	3.46875	0.925225	67° − 42′.2
$1/\sqrt{10}$	1/10	1.90308	0.96056	73° − 51′
1/4	1/16	1.542954	0.97325	76° − 43′ 15″
1/5	1/25	1.33815122	0.98195	79° − 06′

Angular position of the crank arm when the piston velocity is maximum for values of r/c, crank arm to connecting rod ratio, ranging from 1/2 to 1/5.

Instead of picking isolated values of r/c, and solving as above for θ_m and $\sin\theta_m$ by Graeffe's method applied to (13), we may treat x and θ given by (13) as functions of a variable ρ, as done in the next section, and thus make a study of the variations of x, and the physical quantities dependent on x, under changes in $\rho = r^2/c^2$.

NO. 5. THE ANGLE FOR MAXIMUM PISTON VELOCITY, AS A FUNCTION OF RATIO OF CRANK ARM TO CONNECTING ROD, r/c.

Since (13) gives x as a function of ρ we may find the maximum of x as ρ varies. First note that the *minimum* possible solution of our cubic (13), has the value of unity for $x = \csc^2\theta$ by definition since it is the reciprocal of $\sin^2\theta$. Moreover this minimum of unity is seen at once from (13) to occur in two cases, i.e., when $\rho = 1$, or 0. It will be recalled from section one that these each represent simple harmonic motion, Case (a) when $\rho = 1$ and $r = c$, and Case (b) when $\rho = 0$ and $c = \infty$. The maximum speed for this type of motion is known to occur at the mid point of the corresponding vibration where $\sin\theta = 1$ for the situation given by Fig. 1. These critical values of unity for $x = \csc^2\theta$ in (13) for $\rho = 1$ and 0, are thus seen to check our knowledge of the corresponding conditions.

The *maximum* of $x = \csc^2\theta$, or minimum $\sin\theta$ determined by (13) under variations of ρ, is given by setting the derivative of x in (13) with respect to ρ equal to zero. Thus the derivative of x in (13) is:

$$\frac{dx}{d\rho} = \frac{x - 2\rho}{3x^2 - 2x - \rho}. \tag{18}$$

Setting (18) = 0, we obtain $x = 2\rho$ for the condition giving maximum x; and substituting $x = 2\rho$ in (13), we find what the x and ρ, so deter-

mined, are:

$$\rho = \frac{5}{8} = \frac{r^2}{c^2}; \qquad y = r\sin\theta = \frac{2}{5}r\sqrt{5}; \quad \frac{r}{c} = \frac{\sqrt{10}}{4}, \tag{19}$$

$$x = 2\rho = \frac{5}{4} = \csc^2\theta; \quad \theta = 63°26'.$$

The corresponding values of θ, and $y = r\sin\theta$ are shown on the right. These show the position where the maximum of the piston velocity (2) occurs at its farthest position back around the circle from $\theta = \pi/2$, under changes in the ratio $\rho = r^2/c^2$ between crank arm and connecting rod. (See Fig. 3, and the graph in Fig. 5.) From $y = r\sin\theta = c\sin\alpha$,

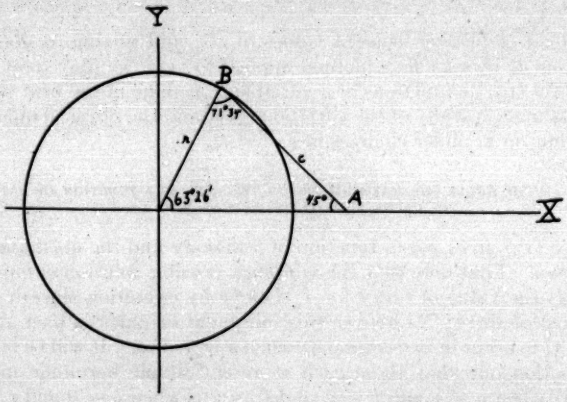


FIG. 3. The minimum angular position (63° 26′) for maximum crosshead velocity with $r/c = 0.791 = \sqrt{5/8}$ and a 45° angularity for the connecting rod.

we have $\sin^2\alpha = \rho\sin^2\theta = 1/2$. That is: $\sin\alpha = 1/\sqrt{2}$; $\alpha = 45°$ at this farthest position of the maximum velocity as shown in Fig. 3, with r, a little more than 3/4 of c; $r = \sqrt{10}/4$.

Note that (19) gives an exact solution of the cubic (13) for the special $\rho = 5/8$, determined by (18) that maximizes x. Equations (18) and (19) point to an easy way of obtaining a series of exact solutions of (13).

This method is to set x equal to an *arbitrary* multiple, n, of ρ instead of twice ρ, as in (19), and then substituting in (13) determine ρ and hence x, y and any other quantities dependent on these critical values. This is, in effect, a trial and error method of solving (13). For, by choosing n arbitrarily (13) determines ρ, which may be made to fit

approximately cases of interest by varying our choice of n. Thus take:

$$x = n\rho. \tag{20}$$

Then from (13)

$$\frac{r^2}{c^2} = \rho = \frac{n^2 + n - 1}{n^3}\ ;\quad x_m = n\rho = \csc^2\theta = \frac{n^2 + n - 1}{n^2},$$

$$\cos^2\theta = \frac{n - 1}{n^2 + n - 1}\ ;\quad \rho\cos^2\theta = \frac{n - 1}{n^3}, \tag{21}$$

$$y_m = r\sin\theta = \frac{nr}{\sqrt{n^2 + n - 1}}\ ;\quad 1 - \rho\sin^2\theta = \frac{n - 1}{n}.$$

TABLE V.

n	$\rho=\frac{r^2}{c^2}=\left(\frac{r}{c}\right)^2$ $\rho=\frac{n^2+n-1}{n^3}$		When piston velocity is maximum				When crank arm and connecting rod are at right angles		
			$x=\csc^2\theta_m$ $=n\rho$	$\frac{y_m}{r}=\sin\theta_m$	θ_m	$\frac{-V_m}{\omega r}=\frac{n+1}{\sqrt{n^2+n-1}}$	$\frac{y_1}{r}=\frac{1}{\sqrt{1+\rho}}$	θ_1	$\frac{-V_1}{\omega r}=\sqrt{1+\rho}$
1	$\rho=1$	$\frac{r}{c}=1$	$x=1$	$\frac{y_m}{r}=1$	$\theta_m=90°$	2	$\frac{1}{\sqrt{2}}=.707$	45°	$\sqrt{2}=1.414$
1.2 =6/5	$\frac{205}{216}$	.972	$\frac{41}{36}$	0.93704	69°34′	1.7179	0.71629	45°44′53″	1.3960
5/4	$\frac{116}{125}$	.960	$\frac{29}{25}$	0.92848	68°12′	1.66782	0.7199	46°2′45″	1.390
4/3	$\frac{57}{64}$	.945	$\frac{19}{16}$	0.917	66°30′	1.60301	0.7272	46°39′	$\frac{11}{8}=1.3750$
3/2	$\frac{22}{27}$	.905	$\frac{11}{9}$	0.9045	64°45′	1.5075	0.74231	47°56′	$\frac{7}{\sqrt{27}}=1.3441$
2	$\frac{5}{8}$	.791	$\frac{5}{4}$	0.894428	63°26′	1.34164	0.7845	51°40′	1.27475
3	$\frac{11}{27}$	.635	$\frac{11}{9}$	0.9045	64°45′	1.20604	0.84292	57°27′	1.1856
4	$\frac{19}{64}$	.543	$\frac{19}{16}$	0.917	66°30′	1.15	0.8781	61°25′	1.14
5	$\frac{29}{125}$	.480	$\frac{29}{25}$	0.92848	68°12′	1.114	0.9009	64°17′	1.10995
6	$\frac{41}{216}$	.435	$\frac{41}{36}$	0.93704	69°34′	1.0932	0.9168	66°27′30″	1.070
8	$\frac{71}{512}$	.374	$\frac{71}{64}$	0.94939	71°42′	1.068	0.9365	69°28′	1.066
10	$\frac{109}{1000}$	.330	$\frac{109}{100}$	0.9578	73°18′	1.0535	0.9445	71°43′	1.0530
11	$\frac{131}{1331}$	.313	$\frac{131}{121}$	0.96107	73°58′	1.0484	0.9539	72°33′	1.04777
14	$\frac{209}{2744}$	.275	$\frac{209}{196}$	0.96838	75°33′	1.0376	0.96393	74°34′	1.03742
17	$\frac{305}{4913}$	.248	$\frac{305}{289}$	0.97341	76°45′	1.03067	0.97031	76°0′9″	1.03059
26	$\frac{701}{17576}$	.1996	$\frac{701}{676}$	0.98201	79°7′	1.01976	0.98063	78°42′15″	1.01975
∞	0	0	1	1	90°	1	1	90°	1

Then from (2) or (3) and (21), we have:

$$V_m = -\omega r \sin\theta \left(1 + \frac{r}{c}\frac{\cos\theta}{\sqrt{1-\rho\sin^2\theta}}\right) = -\omega y_m \left(1 + \sqrt{\frac{\rho\cos^2\theta}{1-\rho\sin^2\theta}}\right)$$
$$= -\omega y_m \left(\frac{n+1}{n}\right) = -\omega r \left(\frac{n+1}{\sqrt{n^2+n-1}}\right). \tag{22}$$

The velocity V and position also are of interest where, for a given ρ, the connecting rod is tangent to the circle of Fig. 1 and at right angles to the crank arm. For this case, we have from (8)

$$y_t = r\sin\theta_t = \frac{rc}{\sqrt{c^2+r^2}} = \frac{r}{\sqrt{1+\rho}} \tag{23}$$

and when (23) is substituted in (3), we have

$$V_t = -\omega y_t \left(1 + \sqrt{\frac{r^2-y_t^2}{c^2-y_t^2}}\right) = -\omega y_t(1+\rho) = -\omega r\sqrt{1+\rho}. \tag{24}$$

With the help of (20) to (24) we may tabulate and compare the critical solutions of (13) and corresponding velocities.

The above tabulated results are perhaps more quickly evaluated if we graph the two velocities and the corresponding angles at which they occur against r/c, the ratio of crank arm to connecting rod.

Figure 4 shows the maximum velocity, V_m, and the velocity where the crank arm and connecting rod are at right angles, V_t, plotted against the ratio of these lengths, r/c. For values of r/c less than 0.5, i.e., for all practical purposes, these differ (in this range) by less than one per cent, and by too small an amount to be shown on the graph, as appears from Table V also.

Figure 5 shows the two graphs of the angular position of the crank arm (see Fig. 1) for the above two velocities. Note that there is a minimum θ_m at 63°26′, where $n = 2$ in Table V, whereas θ_t does not show a minimum. As with the velocity magnitudes there is but little difference in angular position of V_m and V_t over the practical engineering range, which does not include ratios r/c greater than 0.5. This $V_m(r/c = .5)$ occurs less than 5° past the tangent to the circle (Fig. 1) position of the connecting rod where the velocity is V_t. Where r/c is .25 (near $n = 17$, where $\rho = r^2/c^2 = 305/4913$, representing an engineering fair average ratio for r/c of about one to four) this angular difference is less than a degree, and the difference between the velocities (Table V) is only about one part in ten thousand.

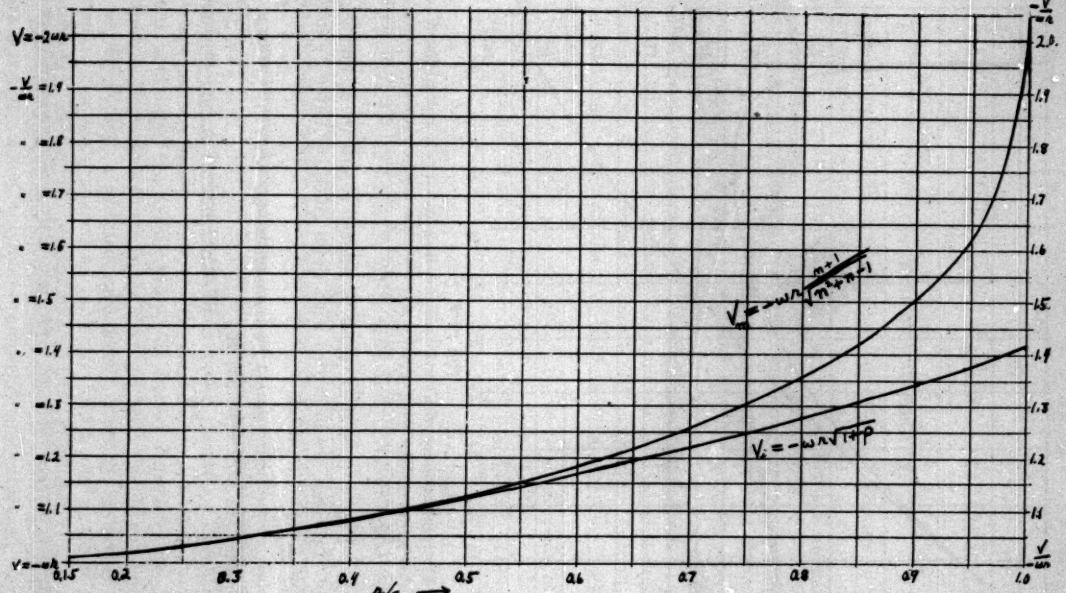


FIG. 4. Maximum crosshead velocity V_m compared with V_t, for which crank arm and connecting rod are at right angles. The horizontal plotting line intervals are 5 per cent. of ωr. Note that for the range of engineering values, $(r/c < .3)$ V_m and V_t differ by less than 1 per cent.

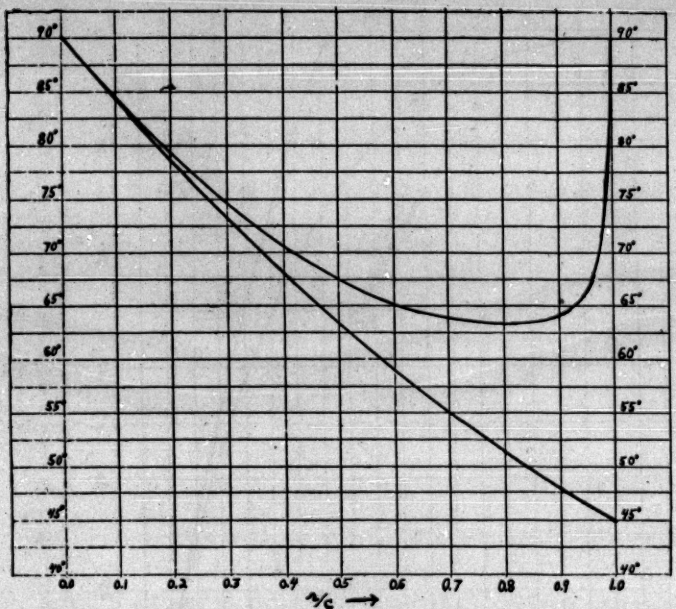


FIG. 5. The angular position of V_m and V_t as r/c ranges from 0 to 1, with minimum for θ_m at 63° 26′ where $r/c = 0.791$ according to Fig. 3. Note that through engineering ranges ($0.15 < r/c < 0.3$) the angular positions of these velocities are only about one degree apart.

Note that Fig. 5 as well as Table V shows values of θ_m and x_m, solutions of (13), that are in accord with those given by Table IV and obtained by Graeffe's method.

The opinions or assertions contained in this article are the private ones ot the writer and are not to be construed as official or reflecting the views of the Navy Department, or the Naval Service at large.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

EXTREMELY SENSITIVE CARBON MONOXIDE INDICATOR DEVELOPED BY BUREAU CHEMISTS.

A colorimetric indicating gel that will detect and estimate less than 1 part of carbon monoxide in 500 million parts of air, a sensitivity more than 100 times greater than attained by former chemical indicators, has recently been announced by the Bureau's Gas Chemistry Section. This gel will also detect 0.001 per cent. by volume in less than 1 minute, and determine physiologically significant amounts (0.01 to 0.4 per cent.) in approximately one minute at ground level. The indicator is so sensitive that it is possible to diagnose carbon monoxide poisoning by analyzing exhaled air instead of taking a blood sample for analysis. Developed to meet the war-time demand for a simple and sensitive means for determination of very small amounts of carbon monoxide in military equipment and installations, it offers promise for many peace-time applications.

Early in the war it was evident that the determination of physiologically significant amounts of carbon monoxide in aircraft, particularly at high altitudes, would assume increasing importance. This was true as well with such equipment as flame throwers, tanks, gun pits and turrets, PT boats, landing craft, and aircraft carriers.

Existing methods of determining carbon monoxide possessed two decided disadvantages. First, the simple and inexpensive, easily available methods were not sufficiently sensitive. Second, those which were sufficiently sensitive were unsuited for field use, with the exception of one expensive instrument, available in limited quantities, whose operation and maintenance were generally beyond the abilities of non-technical men in the services.

During 1941 the Royal Aircraft Establishment, Farnborough, England, had developed an indicator for the detection and estimation of small amounts of carbon monoxide on board aircraft. As this indicator was not entirely satisfactory, the Bureau of Aeronautics, Navy Department, and later the U. S. Army Air Forces requested the National Bureau of Standards to study the RAE gel, with the object of improving it or developing a suitable substitute. Starting with this material Bureau chemists produced an indicator about four times as sensitive more adaptable to field conditions and truly colorimeter.

The indicating material, yellow in color, is a silica gel impregnated with a complex silico-molybdate compound and catalyzed by means of

* Communicated by the Director.

palladium in the form of the sulfate. The yellow indicator turns various shades of green and bluish-green on exposure to very low concentrations of carbon monoxide. The color response is a function of time by concentration of carbon monoxide—a direct parallel to the physiological response to this gas. For use, the indicator gel is sealed in a small glass 5 in. long and the diameter of a lead pencil, with protecting layers of pure silica gel in each end of the tube.

To make a test, the ends of the indicating tube are broken off and the tube inserted in an ordinary 2-oz. rubber aspirator bulb equipped with a special rate-controlling valve. The air to be tested is drawn through the tube by squeezing the bulb once and any color that develops is compared with a set of standard color chips. Such a test can be made by untrained personnel in about one minute. For use in aircraft, tables giving correction factors for altitude are provided. Complete equipment for field use can be carried in a box which fits the pocket of a field jacket.

The NBS Colorimetric Carbon Monoxide Indicators were made only at the National Bureau of Standards, and altogether more than a half million were supplied for the military services of the United States, Canada, and Great Britain. Patents have been issued on the device and assigned to the Secretary of Commerce.

Among the prospective peace-time uses are examination of air in and around busses, automobiles, garages, furnaces and furnace rooms, industrial plants, and civilian transport planes; and in the diagnosis of carbon monoxide poisoning. The tubes may also be modified to serve as detectors of other reducing gases and vapors, including many organic vapors.

EQUILIBRIUM REACTIONS IN THE LECLANCHE DRY CELL DETERMINED BY X-RAY AND MICROSCOPE.

Identification of the compounds present in the Leclanche dry cell by means of X-ray diffraction methods and the petrographic microscope gives a much clearer picture than ever before of the reactions taking place within the cell. At the same time, certain anomalous results that have been previously observed are explained by the discovery that reactions are entirely different in acid and basic solutions.

Although the Leclanche cell (that is, the familiar dry cell best known in the form of flashlight batteries) was invented over 80 years ago, an adequate explanation of its theory of operation has not been previously worked out. There is no general agreement regarding the essential current-producing reaction or the final reaction products. Recent X-ray and microscopic studies, however, have disclosed many characteristics of the oxides of manganese and the carbon blacks, essential components of the cells.

In this paper, H. F. McMurdie of the Bureau's Laboratory of Microscopic and Diffraction Analysis, with D. N. Craig and G. W. Vinal of the Electrochemistry Section, approaches the problem in three parts. First, they identify the solid phases formed with a solution similar to a dry cell electrolyte under equilibrium conditions, but with no manganese compounds present (systems containing ZnO, $ZnCl_2$, NH_4Cl, H_2O).

The second part of the paper deals with manganese oxides and the rôle of soluble divalent manganese in typical dry-cell electrolytes. When manganese dioxide is reduced in the presence of zinc, a condition existing in the dry cell, the zinc-manganese compound, hetaerolite ($ZnO.Mn_2O_3$), is formed. This compound is also identified as a product of discharge in dry cells.

In the third part the pH vs. potential relationships of manganese dioxide electrodes are described. The nature of the reactions in acidic and basic solutions is shown to be entirely different, the valence change in the former being two, and in the latter one. In typical dry-cell electrolytes and over the pH range usually encountered in dry cells, the data indicate that the most important equilibrium reaction is that resulting in the formation of hetaerolite.

Both X-ray and chemical determinations agree in establishing the presence of small amounts of soluble manganese and the character of the oxide crystals at any stage between low and high pH values.

LOW TEMPERATURE DRY CELLS.

Dry cells that perform satisfactorily at temperatures of −30° C. (−22° F.) and lower, and that also possess other essential properties, such as suitable shelf life, are described in a paper by G. W. Vinal, Earl Otto, and C. K. Morehouse.

As early as 1922, Bureau scientists had pointed out that dry cells of the ordinary ammonium chloride type become inoperative at a temperature of about −20° C.(−4° F.). Operations in arctic regions and at high altitudes require a cell that will operate at considerably lower temperatures, and up to 1941, dry cells meeting this requirement were not available in the United States. Accordingly, the Bureau was requested to undertake the development of a low-temperature dry cell.

On the basis of previous unpublished work at the Bureau, a cell was developed by substituting methylamine hydrochloride for ammonium chloride; the electrical characteristics were good, but its shelf life was poor.

Further investigation revealed that the addition of a small amount of ammonium chloride caused a stabilization of voltage, amperage, and capacity, and prolonged the shelf life.

Following the Japanese Manchurian campaign in 1931, two Japanese scientists published results of an investigation of calcium chloride-zinc chloride solutions, in which they found that some of these solutions did not freeze even at −100° C. Although they succeeded in producing a battery that would function at low temperatures, it did not prove entirely satisfactory.

Extending this work at the Bureau it was found that certain formulas had apparently been overlooked. For example, cells in which the mol strength of the zinc chloride is equal to or greater than the calcium chloride show a very constant voltage. Addition of ammonium chloride increases its output.

The calcium chloride-ammonium chloride type of cell has given outstanding performance and, at present, seems to be superior to the methylamine hydrochloride type.

PLASTIC COATING PROTECTS DRY CELLS.

A vinyl-resin plastic coating approximately 1 mm. thick will protect dry batteries from deterioration and extend their shelf life at 130° F. for periods ranging up to 6 months, as shown in accelerated tests conducted by W. J. Hamer, J. P. Schrodt, and G. W. Vinal. Storage in glass, copper cans, and metal-lined bags increased shelf life slightly, but batteries sealed in glass without venting relief built up such pressures that they exploded. At this temperature unprotected batteries were unserviceable at the end of 2 months.

Ordinarily dry cells are stored at moderate temperatures (68° to 78° F.) but during the war batteries were shipped and stored at temperatures which in some cases reached 130° F. Under these conditions batteries deteriorated rapidly, and this study was undertaken to ascertain the reasons for the deterioration and to provide means for preventing it. BA-38 dry batteries, "D-size" flashlight cells, and sections of buoy batteries consisting of "J-size" cells were used for these tests.

At 130° F. all of the batteries were found to gas with evolution of hydrogen. Samples of the gas were withdrawn at intervals and analyzed with the mass spectrometer, a procedure that permitted hundreds of analyses that could not otherwise have been made. Composition of the gas samples indicated the process taking place within the containers, as follows: (1) in copper cans and metal-lined bags, the gas is essentially air because leaks develop in the containers as the pressure builds up within them; (2) the gas in glass apparatus eventually becomes pure hydrogen; (3) the gas in plastic containers approaches an equilibrium composition because of the permeability of the plastic to gases. In other words the permeability of the plastic permits the escape of some of the gas formed, and access to air and water vapor—the batteries may literally "breathe." Other data indicate that drying-out of the bat-

teries and possibly the oxygen content of the surrounding gas contribute largely to the rapid deterioration of dry batteries at elevated temperatures.

POISSON'S RATIO OF STRUCTURAL ALLOYS DETERMINED UNDER LARGE TENSILE STRAINS.

Metals under load exhibit characteristic elastic properties up to a certain point; higher stresses produce plastic properties. Metals used in structures are generally under such loads that they behave elastically. However, as more exact knowledge of the properties of materials has become available, factors of safety have been decreased and higher stresses, leading into the plastic range, have come into use. This requires further study, then, of the plastic properties of metals and their behavior in this range.

Among the properties on which very little data are available, for any materials, is the relation between stretch of the metal and the accompanying lateral contraction, known as Poisson's ratio. Methods for finding these values in the elastic range and in the plastic range, for stretches up to 18 per cent., are described in the October *Journal of Research* (RP1742). Developed by Ambrose H. Stang, Martin Greenspan, and Sanford B. Newman, of the Bureau's Engineering Mechanics Laboratory, they have been applied to some aluminum alloys and structural steels, and these results are also reported in the paper.

The values of axial stretch vs. Poisson's ratio and axial stretch vs. tensile stress for more than 100 specimens were obtained. The materials investigated were aluminum alloy sheets of 24ST and 24SRT, of various thicknesses, up to 0.125 in.; chrome-molybdenum steel 0.25 in. thick; and structural steels from 0.375 in. to 1.5 in. in thickness. Considerable variation was noted in the values of Poisson's ratio for the same stretch of nominally identical specimens.

It was found that the value of Poisson's ratio increased rapidly from its value under elastic conditions, after the proportional limit had been passed, to a maximum at a stretch of from 3 to 6 per cent., and then gradually decreased as the strain was increased. The maximum value was always definitely less than one-half, the value predicated on purely plastic conditions. The increase in volume due to tensile load was also calculated from some of the test results. It was found that the volume continued to increase with stretch, within the limits of the test, and did not become constant as is to be expected for purely plastic materials.

These data will also be useful when strain analyses are made of the materials which have been subjected to two-dimensional stress conditions in the plastic range. The use of actual rather than assumed values for the strains in different directions should contribute evidence toward the validity of the various theories of strength of metals.

VOLUME CHANGES IN CONCRETE DURING FREEZING AND THAWING OBSERVED BY NEW DILATOMETRIC METHOD.

Moist concrete, when cooled slowly below the freezing point of water (32° F.), may reach temperatures as low as 21.5° F. before the water within the concrete freezes, but this supercooling is followed by abrupt increases in volume and temperature, as shown by recent investigations at the Bureau.

Concrete exposed to weather and to low temperatures is subject to damage by freezing and thawing; this damage is due in part to volume changes that take place during the process. In order to determine the effects of the rates of freezing and thawing upon the volume changes (transient and permanent) of concrete, a method has been devised by Rudolph C. Valore, Jr., of the Concreting Materials Section, for making continuous observations of the volume changes, the first time that such a method has been successfully used.

In this method, volume changes are indicated by the displacement of mercury from a steel dilatometer into a burette graduated in hundredths of a milliliter. The cylindrical test specimen (2 by 7.25 inches), with a thermoelement at its center, is sealed into the dilatometer, and the entire assembly immersed in a liquid bath in which the temperature cycle of +40° to −20° to +40° F. is traversed at various rates.

For air-dried concrete specimens the volume changes were found to be uniformly linear and independent of the rate of temperature change. Coefficients of linear thermal expansion of 6.0×10^{-6} per °F. were indicated for a concrete containing siliceous sand and $\frac{3}{8}$ inch gravel, and 6 bags of portland cement per cubic yard of concrete.

In moist specimens of the same composition, departures from the linear volume-temperature relationship were ascribed to freezing or thawing of water within the capillary structure of the concrete. The contraction of a specimen during cooling was interrupted as the water froze, and expansion during heating was retarded as ice melted.

When cooled slowly (10° F. or less per hour), moist specimens showed uniform contraction to temperatures as low as 21.5° F., or 10° F. below the normal freezing point of the water in the concrete. Following this apparent supercooling effect there was an abrupt increase in volume and temperature as water within the specimen began to freeze. When the temperature was gradually lowered after freezing had begun, the specimen again contracted uniformly, but at a greater rate than before freezing. In most cases the total contraction during the slow cooling from +40° to −20° F. was greater for the moist than for the air-dried specimens, despite the abrupt increase in volume in the moist specimens as freezing began.

The magnitude of this increase depended mainly upon the degree of moisture saturation of the concrete, the extent to which the specimen had been weakened by previous freezings, and the temperature to which

the water in the specimen could be supercooled before freezing. It was found that the degree of supercooling could be controlled by varying the period of time between freezing and thawing cycles during which the specimen remained unfrozen. Supercooling was virtually eliminated when this "recovery" time before a cycle was reduced to a minimum, but it was at a maximum when this time exceeded 100 hours.

During the fastest cooling, +40° to −20° F. in 30 minutes, moist specimens showed smaller amounts of contraction than during the slow cooling. These contractions became smaller as the specimen was weakened with each succeeding fast cycle. In addition, a rapidly cooled specimen maintained at −20° F. showed shrinkage lasting for several days before reaching constant volume, whereas, when cooled sufficiently slowly, the specimen volume at any temperature during a given cycle was an irreducible minimum.

The volume of a moist specimen was usually greater at the end of a cycle of freezing and thawing than at the start. The amount of this residual expansion for each cycle varied directly with the rate of cooling, the degree of supercooling, the number of cycles previously undergone by the specimen, and the degree of saturation of the concrete. The residual expansion varied inversely with the rate of the thawing.

On the basis of residual expansions, the combination of the fastest freezing and the slowest thawing appeared to be most destructive; and the slowest freezing (with a minimum of supercooling), combined with the fastest thawing, the least destructive of the treatments to which these specimens were subjected.

No attempt was made to analyze stresses on the basis of transient volume changes and elastic properties for moist specimens, since stresses so computed would be, in some cases, several times greater than the nominal tensile strength of the concrete.

ELECTRODE FUNCTION (pH RESPONSE) OF POTASH-SILICA GLASSES.

A study of the electrode function (pH response), of a limited series of potash-silica glasses, indicates voltage departures from the theoretical, at pH values greater than 40, of such magnitude as to render them unsuitable for glass electrodes, according to a report in the October *Journal of Research* (RP1743). Furthermore, results of the investigation support the theory that glasses of high hygroscopicity (water sorption) will have sensitive pH responses, and that changes in the durability of a glass will be accompanied by voltage departures for an electrode prepared from such a glass.

These two properties, namely hygroscopicity and chemical durability in solutions of a wide range of pH values, appear to determine largely the suitability of a glass for measuring the hydrogen-ion activity of aqueous solutions, in accord with the simplified Nernst equation,

$E = 0.000198T$ pH. This equation yields a theoretical value of 59 millivolts per pH at 25° C.

Five potash-silica (K_2O-SiO_2) glasses were used in this investigation by Donald Hubbard, which included determinations of hygroscopicity, pH response, and chemical durability. These glasses had been carefully prepared in the Bureau's Glass Section for studying their physical properties. Their composition varied from 71.83 to 83.07 per cent. SiO_2.

The glass electrodes were blown as thin walled tubes on the end of tubing made from the experimental glasses, and the inner electrical connection was obtained by filling the bulb with mercury. Chemical durability measurements were made by the interferometer method, on a single specimen of glass that was reground and repolished after each test. In order to have results comparable with work previously reported in other publications the Britton-Robinson universal buffer solutions were used.

All of the glasses investigated showed a very high hygroscopicity, with an accompanying sensitive pH response. However, between pH 2 and 4, glasses of 71.8 and 73.6 per cent. SiO_2 gave pH-response values higher than the 59 millivolts per pH predicted from the Nernst equation. Electrodes prepared from all of the specimens demonstrated large voltage departures from the straight line relation with increasing alkalinity of the buffer solutions.

For a glass of 75.56 per cent. SiO_2 content, the voltage departures were compared with the sodium-ion concentration, $[Na^+]$, of the buffer solution, as well as with the chemical durability of the glass. The voltage departures correlated with the magnitude of the attack more convincingly than with pNa.

Although their hygroscopicity is high, giving them a very sensitive pH-response, the durability characteristics of these potash-silica glasses are so poor that voltage departures appear at pH values greater than 4.0, even for the best glass examined.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

DENNIS, WILLARD KELSO. Recent Aeronautical Literature. 1946.
FORESCH, CHARLES, AND W. PROKOSCH. Airport Planning. 1946.

ASTRONOMY.

DAVIDSON, MARTIN. From Atoms to Stars. 1946.

BIOCHEMISTRY.

Annual Review of Biochemistry. Volume 15. 1946.
COHN, EDWIN J., AND J. T. EDSALL. Proteins, Amino Acids and Peptides. 1943.
GREEN, DAVID E. Currents of Biochemical Research. 1946.

BIOGRAPHIES.

FORD, EDWARD. David Rittenhouse. 1946.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

ANDERSON, JOHN ANSEL. Enzymes and their Role in Wheat Technology. 1946.
BOWDEN, SYDNEY THOMAS. Phase Rule and Phase Reactions 1945.
D'ALELIO, GAETANO F. Experimental Plastics and Synthetic Resins. 1946.
DEMING, HORACE GROVE. General Chemistry. 1946.
ELLIOT, STANLEY. The Alkaline-Earth and Heavy Metal Soaps. 1946.
GLASSTONE, SAMUEL. Textbook of Physical Chemistry. Second Edition. 1946.
KARREN, PAUL. Organic chemistry. 1946.
MANTELL, CHARLES LETNAM. Industrial Carbon. 1946.
Modern Plastics Catalog. 1946.
NORD, F. F. Advances in Enzymology and Related Subjects. 1946.
WAIT, DOUGLAS. The Manufacture of Enamel Paints. 1946.
WHALLEY, MURIEL E. Abstracts on Synthetic Rubber. Parts Three and Four. 1946.

DIRECTORIES.

Who's Who in America. 1946–47.

ELECTRICITY AND ELECTRICAL ENGINEERING.

Coyne Electric School. Electronics for Electricians and Radio Men. 1945.
ESTY, W., R. A. MILLIKAN, W. L. MCDOUGAL. Elements of Electricity. 1944.
Federal Telephone and Radio Corp. Reference Data Book for Radio Engineers. Second Edition. 1946.
HYLANDER, CLARENCE J., AND R. HARDING. An Introduction to Television. 1946.

ENGINEERING.

CROFT, TERRELL WILLIAMS. Steam Power Plant Auxiliaries and Accessories. 1946.

MARINE ENGINEERING.

HANSEN, EMIL M. Modern Marine Pipefitting. 1942.
SWANSON, W. E. Modern Shipfitter's Handbook. Second Edition. 1941.

MATHEMATICS.

British Association for the Advancement of Science. Mathematical Tables. Volumes 8 and 9. 1940.
U. S. Works Project Administration. Tables of Langrangian Interpolation Coefficients. 1944.

MECHANICAL ENGINEERING.

CHAPMAN, W. A. J. Workshop Technology. Volumes 1 and 2. 1945.

METALLURGY.

SIMONDS, HERBERT R. Finishing Metal Products. Second Edition. 1946.

METEOROLOGY.

STEWART, JOHN Q. Coasts, Waves, and Weather. 1945.

MINERALOGY.

SPENCER, L. J. A Key to Precious Stones. 1946.

NAVAL ARCHITECTURE.

LORD, LINDSAY. Naval Architecture of Planning Hulls. 1946.

OPTICS.

BORN, MAX. Optik. 1933.
ZWORYKIN, VLADIMIR KOSMA. Photocells and their Application. Second Edition. 1945.

PETROLEUM.

FUCHS, WALTER MAXMILIAN. When Oil Wells Run Dry. 1946.

PHARMACY AND PUBLIC HEALTH.

American Medical Association. New and Nonofficial Remedies. 1946.
KUNZE, RUDOLP. Lecithin. 1941.
SILVER, JOHN A., AND R. CLARKSON. Manufacture of Compressed Tablets. 1944.

PHYSICS.

BORN, MAX, AND PASCUAL JORDAN. Elementare Quantum Mechanik. 1930.
BRILLOUIN, LÉON. Wave Propagation in Periodic Structures. 1946.
COOPER, HERBERT J. Scientific Instruments. 1946.
GAMOW, GEORGE. Mr. Tompkins Explores the Atoms. 1945.
HOUWINK, R. Elasticity, Plasticity and Structure of Matter. 1940.
SCHRÖDINGER, ERWIN. Statistical Thermodynamics. 1946.

RAILROADS.

VAN METRE, THURMAN WILLIAM. Trains, Tracks and Travel. Seventh Edition. 1946.

SCIENCE.

COKER, WILLIAM CHAMBERS. Studies in Science. 1946.

SUGAR.

Cuba Economica y Financia. Anuario Azucarero de Cuba. 1945.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

A Reprint Collection as an Easily Accessible Source of Reference Material.—ANNE HIMMELBERGER LONGENBACH, Librarian. The research material needed by the members of the Biochemical Research Foundation is extremely varied. Biochemical investigation by its very nature involves reference to many of the different branches of science, —chemistry, physics, electronics, cytology, pharmacology, therapeutics, etc. In order to have technical reference material connected with all these phases of biochemistry quickly available a comprehensive scheme is necessary.

The reprint collection is the pith of the research library of the Biochemical Research Foundation. The number of reprints on September 1, 1946 was 33,650. The value of scientific material in reprint form lies in its ease of access. Locating the article and getting it quickly are often of prime importance. Articles in reprint form are easier to handle: when ten or twenty or more articles are to be consulted, they are easier to assemble and transport, even from the library to the laboratory room, than ten or twenty bound journals or books. When the paper desired is sought in the bound journal, that particular number of the journal may be in use by someone else for an entirely different article; this delay and inconvenience is obviated if the paper is in reprint form. Also, no matter how extensive the list of journals received by the library, there must be some limit to the journals received; but there is no limit to the number of journals from which reprinted articles may be obtained.

The reprints in our library are obtained through the courtesy of authors and institutions, through exchange mailing lists, and in response to our printed postcard request.

The time expended in making research material in reprint form easily accessible, amounts to about half of each day by one member of the library staff. The reprints must be checked for the possible possession of another copy, perhaps previously received from a co-author, in which case copies 2, 3, etc. receive the number carried by the first copy. For each newly received reprint a card is made for every author and every subject; 3 × 5 cards are used.

The author and the subject files are in separate cases, but, after years of constant use, it seems that a minute fraction of time and effort might be saved if they were combined so that there would be just the one catalog.

The reprints are numbered consecutively as they are received. When papers appear in a series, we use the same number and each

reprint number and card number then carries the added roman numeral I or II or III. The number is placed in the upper right corner of each reprint and of each author and subject card. The cards are filed immediately; the incoming reprints are displayed on a slanted reading shelf for several days and are then placed in a steel cabinet. They are now accessible upon minimum expenditure of time and effort not only to the Library Staff but to the members of the Research Staff as well. Since the reprints stand in the steel cabinets in numerical order there is no difficulty in locating the article desired.

The drawers of our steel cabinets are filled with double rows of reprints, and material larger than 7 × 10 has the code letter A added to the number and is filed in cabinets also marked "A". Because the reprints are filed numerically they can of course be placed on shelves or in boxes. We have found, however, that the four-drawer steel cabinets afford good protection and that the number in this position on each reprint is easy to see.

No attempt is made to have together all reprints of one author. If all the papers on a certain subject by one author are desired, the numbers of the reprints are taken from the author file and the reprints are located by number. Nor is any attempt made to have subjects grouped or adjacent. This would be very complicated, if not hopeless, when several subjects were involved and would serve no special purpose since the cards in the subject file show the numbers of the reprints and the reprints are immediately available in numerical order.

Upon using this comparatively simple but quite adequate system for even a short time, it becomes apparent that material other than scientific articles in "reprint" form can also be numbered, filed, and easily found. Many a piece of ephemeral material, that bug-a-boo of the technical librarian of today, has been included in our reprint collection. Unpublished research work, negative results, periodical reports of our own, are made easily accessible by this system.

Our reprint library requires much less space, time, and effort to make available scientific material to the research man than would be required if the material were available only in journal or book form. When scientific material becomes available on Micro-cards as proposed by Fremont Rider in "The Scholar and the Future of the Research Library" (Hadham Press, New York, 1944), our reprint collection will be replaced by a card library at a decided economy of space, time and effort. Until that time, however, the reprint collection will continue to serve as a major source of research material at the Biochemical Research Foundation.

BOOK REVIEWS.

LE COBALT, by Robert Perrault. 151 pages, drawings, tables and illustrations, 16 × 25 cm. Paris, Dunod, 1946. Price 390 francs.

For many years cobalt had been used almost entirely as a coloring agent in the ceramic and enamelling industries, but with the discovery, several years ago, of the valuable properties which it displays in metallurgical applications, the metal assumed a new importance. In the present volume Perrault offers a broad survey of this interesting metal.

A brief history of cobalt is followed by a description of the various minerals in which it is found, the location of important deposits is mentioned with the indication that Belgian Congo and Rhodesia produced nearly two-thirds of the world's supply in 1938. Other important deposits are in French Morocco and Canada. Several methods of extracting the metal are given and flow diagrams illustrate these processes.

The various properties of cobalt, physical, mechanical, magnetic, chemical, and physiological form the subject of another chapter.

Numerous usages are discussed with emphasis on cobalt's employment as an alloying constituent in steels and other alloys. It is also used in electroplating and in the manufacture of carbides, and as a catalyzer is proving useful in the production of synthetic petroleum. Experiments have been made on its value as a factor in animal nutrition. The original usage as a coloring agent is also considered.

A final chapter on economic considerations and a bibliography complete the volume.

Designed not only for miners and metallurgists, it is addressed to all those industries which use cobalt in any form. They should find it a valuable resumé of the important data relative to this metal.

GEORGE E. PETTENGILL.

FUHRER DURCH DIE SCHWEIZERISCHE DOKUMENTATION, Herausgegeben von der Schweizerischen Vereinigung für Dokumentation. 2 verm. Auflage. 80 pages, 1946. Price 4.50 Swiss francs.

This second edition of a "Guide to Documentation in Switzerland" was made necessary by the rapid exhaustion of the first edition published in 1942. During the war, Swiss research workers found the "Guide" of immense value since it lists collections containing literature not available in the large Swiss scientific libraries and which could not be included in the General Catalogue of the Swiss National Library. With the vast destruction of libraries in European countries, it was felt that Swiss library resources would assume even greater value, and the reissuance of the guide was even more vital.

The new edition lists 227 documentation centers arranged in subject order. Information given includes address, number of volumes and periodicals, special holdings or subject interests, and regulations governing use of the library. Name, subject and place indexes, together with a list of users of the decimal classification, facilitate use of the "Guide."

The introduction contains six brief articles relating to various phases of documentation. The activities of the Swiss Association for Documentation and the Swiss Association of Librarians are described. Next is an account of the scope and importance of the work done by the General Catalogue and Information Service of the Swiss National Library, which corresponds to the union library catalogues in this country. The last three articles comprise a vocabulary of documentation terms in French, German and Italian, a description of Swiss publications on the standardization of documentation, and a bibliography, and decimal classification.

This small but useful publication gives further evidence of the value and necessity for increasing cooperation in the administration of documentation activities.

GEORGE E. PETTENGILL.

PUBLICATIONS RECEIVED.

Electrical Transmission in Steady State, by Paul J. Selgin. 427 pages, drawings, 14 × 22 cms. New York, McGraw-Hill Book Company, Inc., 1946. Price $5.00.

Applications of Germicidal, Erythemal and Infrared Energy, by Matthew Luckiesh. 463 pages, drawings and illustrations, 14 × 21 cms. New York, D. Van Nostrand Company, Inc., 1946. Price $5.50.

Chemistry of Familiar Things, by Samuel Schmucker Sadtler. Eighth Edition, Revised. 310 pages, illustrations, 14 × 21 cms. Philadelphia, J. B. Lippincott Company, 1946. Price $4.00.

The Electronic Engineering Master Index, Jan. 1935 to June 1945, edited by Frank A. Petraglia. 209 pages, 18 × 26 cms. New York, The Macmillan Company, 1946. Price $6.00.

CURRENT TOPICS.

Lamp Enables Two-Way Talk Over Invisible Searchlight Beam.—Existence of a "talking lamp," which emits infrared radiations enabling secret two-way conversation over an invisible searchlight beam, was disclosed recently by the Westinghouse Lamp Division.

The source of the unseen radiations is caesium vapor, a metal never used extensively in lamps. Not only an efficient generator of infrared, caesium also is a poor visible illuminant, qualifying it for confidential telephonic assignments. Its low visibility, moreover, minimizes requirements for a filter to block out stray visible light which could betray the presence of the signal system.

"With this lamp," explained its designer, Dr. Norman C. Beese, Westinghouse research engineer, "one is able to transmit words practically instantaneously with true telephone quality and at normal conversational speed. In blinker signalling, a maximum of only six words a minute is attainable because words must be spelled out slowly with a shutter system, either by means of visible light or by invisible infrared signalling."

The caesium vapor lamp was designed by Dr. Beese at the request of the United States Navy, which intended to use it for convoy duty and for issuing troop landing directions. A feature particularly attractive to the Navy is that, unlike radio, there can be no eavesdropping or jamming of infrared "beamcasting." Jamming would require the use of a "shutter" device within the limited 25-degree beam and the message is restricted to listeners within the beam spread, only five miles across at its widest point 10 miles away.

By V-J Day, about 3,500 of the 100-watt lamps, made in the manufacturing department pilot plant, had been shipped to the Navy but the auxiliary equipment was not obtained in time for use in combat, Dr. Beese said he was advised. Suggesting peacetime uses, Dr. Beese said the lamp is expected to prove useful in confidential ship-to-shore communications where radio wavebands might be objectionable; in conveying messages among pilots flying through radio "blackouts" in close formation or a few miles apart; or in disaster areas where telephone lines are cut and climatic conditions make radio broadcasting impossible. Infrared beamcasting is unaffected by static and all weather except extremely soupy fog or smoke, the scientist added.

Filling a gap between radar and "walkie-talkie," infrared beamcasting is similar to radio broadcasting except that the sound is carried over ultra-short wave lengths rather than long wave lengths, Dr. Beese pointed out. The waves, transmitted over a carrier wave having a frequency 350 million times the normal broadcast band, have a distance or horizon limitation similar to that encountered in television.

The lamp itself serves as the transmitter. When mounted on a ship's mast in a parabolic searchlight-like reflector, it picks up words spoken into a microphone from the ship's pilothouse or deck and provides wings for the voice to reach the receiving station on another ship or a shore station. At the receiving station, a photoelectric cell mounted in another parabolic reflector picks up the

infrared rays, and with suitable amplification converts them into a reproduction of the spoken words.

The key to broadcasting with light beams, Dr. Beese explained, is the ability of the lamp to alternately dim and brighten, thousands of times a second, a requirement necessary in order to truly reproduce by wave lengths the varying tonal qualities of the human voice, which range in pitch, or frequency, up and down the musical scale.

In this characteristic, called modulability, the caesium vapor lamp has maximum efficiency, reaching a peak of 100 per cent. at some points in the entire usable audio-frequency range of 200 to 3,000 cycles a second. By contrast, the scientist added, a 60-watt household lamp can be modulated to a maximum of only one-tenth per cent.

The "talking lamp" accepted by the Navy is 13 inches long overall and produces its radiation from an arc stream 3 inches long and 1¼ inches in diameter. It is filled with argon gas and caesium vapor. To conserve heat, the inner bulb is mounted within an outer bulb and the intervening space evacuated. The outer bulb, two inches in diameter, is banded with padded metal strips between two ridges in order to maintain accurate alignment and support.

Life tests show that the lamp can operate more than 100 hours without replacement. It burns on direct current electricity but the arc stream itself is modulated by alternating current, superimposed.

Various sizes and shapes of lamps, ranging from 15-watts to 600-watts, were investigated in the research department and were made for both the Navy and the Army Air Forces. The A.A.F. advised that, for inter-plane communication, a 40-watt lamp appeared to be most useful.

Active experimental work at Westinghouse on the caesium lamp started in April, 1944. A bulb was produced in a few months and experiments conducted at the Westinghouse Research Laboratories in East Pittsburgh showed it to be successful.

R. H. O.

NACA Twelve Foot Low Turbulence Pressure Wind Tunnel.—In this tunnel at Ames Aeronautical Laboratory, Moffett Field, California, full scale flight conditions can be more nearly simulated than in any other wind tunnel in existence.

Designed to operate at air speeds up to the speed of sound, the tunnel is unique in providing air flow of negligible turbulence such as is experienced in free atmosphere flight. Further, by pressurizing the air in the tunnel, data directly applicable to airplanes as large as full-scale attack bombers or small transport aircraft can be obtained. The tunnel size and shape were chosen for complete detailed models. The Twelve Foot Pressure Wind Tunnel is the first ever built to combine the features of very high speed, low turbulence airflow, pressurization to simulate full-scale conditions, and large size to accommodate complete models.

In the anti-turbulence sphere which gives the tunnel its unusual shape lies the secret of achieving the unparalleled low turbulence in such a large high speed tunnel. The sphere, located upstream of the test section, contains eight fine mesh screens, each some sixty-three feet in diameter. These screens, higher than a six story building, act to smooth out eddying motions in the air

and produce an orderly flow. Further, the 25:1 contraction ratio from the bulge to the test section acts literally to squeeze the remaining turbulence out of the air stream. Turbulence of the flow in the test section is consequently reduced to the degree that flow conditions may be considered to approximate those normally encountered in free air. Virtual elimination of turbulence error marks a unique advance in wind tunnel technique and usefulness in that there is no way of completely correcting for turbulence, should it exist, in application of data to free flight conditions.

One of the most important considerations in attempting to duplicate flight conditions in a wind tunnel is the combined effects of airplane size, velocity, and density of the air. The combination of these factors, each having a known magnitude, results in an index referred to as Reynolds number. In general, the criterion for similar flow in comparing wind tunnel data with flight data is equality of Reynolds number. In this connection, the ability to vary pressure of the air circulated in the tunnel from ¼ to 6 atmospheres results in the direct control of air density, thereby allowing independent variation of Reynolds number without changing model size or air velocity. This is an important feature for many reasons. For example, by compressing the air to six times atmospheric density, a model having a 10 ft. wing span may be investigated and data obtained which are directly applicable to an airplane with a 60 ft. wing span, or one the size of an attack bomber.

Variation of density in the tunnel from ⅛ to 6 times atmospheric density permits a wide range of tunnel air velocities with a given power input. At pressure below ½ atmospheric, an air-speed of 750 miles per hour is attainable.

Some of the details of the tunnel are tunnel length 336 ft., diameter of working section 12 ft., maximum diameter (in anti-turbulence sphere) 68 ft., contraction ratio 25:1, number of fans 2, diameter of fans 22 ft. 8 ins., number of blades in front fan 12, number of blades in rear fan 20, drive shaft 30 inch diameter hollow steel, main drive power 11,000 h.p., main drive speed 490 r.p.m.

Three multi-stage compressors, capable of handling a total of 25,000 cubic feet of air per minute, are employed to compress and evacuate the tunnel air. Less than two hours are required to raise the density to six atmospheres.

A motor-operated blow-down valve releases the pressurized air when tunnel pressure reduction is desired. Instead of venting this pressure energy to the atmosphere, however, the NACA is constructing an intermittent-type supersonic wind tunnel to utilize air released from the Twelve Foot Tunnel. For the same reason that the "parent tunnel" yields high Reynolds number data, so will the opportunistically constructed supersonic wind tunnel provide data at large scale.

From the viewpoint of technological advancement in aeronautics, the potentialities of the Twelve Foot Tunnel are large. The success of the National Advisory Committee for Aeronautics laminar-flow airfoil developments, as manifested in the high speed performance of the North American "Mustang" and the Lockheed "Shooting Star," represents but a partial exploitation of the laminar-flow principles as a result of limited research facilities. The tunnel provides the only research facility in which high speed laminar-flow wings and bodies can be fully investigated and developed. It will be invaluable too for the full study of subsonic and transonic characteristics of supersonic airfoils and control surfaces. The tunnel will also be used to explore fully the potenti-

alities of propellers, which have a definite future in the high speed, high altitude field. Critical design parameters for inlet ducts and shapes for large airflow volume engines, such as turbo-jets and ram-jets, are natural fields of investigation for the tunnel. Finally, complete aircraft configurations will be investigated for further knowledge of criteria for tomorrow's high performance aircraft.

R. H. O.

Miniature Ultraviolet Lamp Developed for Home Refrigerators.—Development of an ultraviolet lamp which will bring an electronic sentinel of sanitation into home refrigerators for a nickel a month was announced by the Westinghouse Lamp Division.

The new lamp is a midget Sterilamp the size of an automobile headlight bulb, the first bactericidal ultraviolet source ever designed in other than the tubular shapes which stood guard over food processing plants during the war.

Screwed into a niche of the refrigerator cabinet's interior, the new lamp simultaneously pours forth a barrage of bacteria-killing rays and produces ozone in the cabinet's air. It is recommended for both high humidity and low humidity types of mechanical refrigerators, said Charles W. Flood, Jr., manager of the Westinghouse Sterilamp-Tenderay department. Mr. Flood added:

"Our research proves that the bactericidal radiation and ozone wave lengths emitted by this lamp assure odorless refrigerators, improve sanitation, enable longer preservation of food in its original stored state and check the growth of mold and bacteria on food.

"Ozone, an air purifying form of oxygen present in outdoor air immediately after a thunderstorm, retards the 'odor swapping' which can adversely affect the taste of butter, milk and other foods sensitive to cross contamination."

Resulting from a two-year investigation conducted by Chalmers Morehead, Westinghouse engineer, the lamp produces the bactericidal and ozone-creating wave lengths of ultraviolet in the controlled amounts known to be necessary through almost 10 years of experiments with domestic refrigerators.

The ozone, Mr. Flood explained, diffuses rapidly throughout the entire cabinet, purifying the food vapors by breaking down the vapor molecules. The ultraviolet destroys air-borne bacteria, viruses, mold and food odors within the range of its radiation.

The lamps are indispensable to high humidity cabinets, retarding spoilage and mold growth which otherwise soon would occur under the higher temperatures and higher moistures desired to preserve surface color, flavor, texture and juiciness of uncovered foods. In low humidity cabinets, the types in greatest demand prior to the war, there was little difficulty from bacterial or mold spoilage because of the low average humidity and low temperature during most of the refrigerator's operation but food odor accumulations always have been a problem, Mr. Flood said.

R. H. O.